Skills, Informality and the Size Distribution of Fims. *

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> The size distribution of firms *"is a topic on which issues of economic policy are held to hinge: in wealthy economies, 'bigness' is very widely viewed as a menace against which government activity should, perhaps, be directed; in poor economies, 'littleness' as a sign of backwardness dealt with by government policy. The lack of discipline with which such issues are typically discussed reflects, I think, the lack of an adequate understanding of the forces determining firm size"[Lucas(1978)]*

Latin America and many developing countries in the rest of the world are plagued with many small unproductive firms. These firms often do not pay taxes and are part of the *informal* sector. The two issues are tightly connected as it doesn't pay for the tax authority to put many resources into auditing small firms.

There has been a lot of academic and policy interest in studying whether size dependent policies and other distortions that favor small businesses or prevent them from growing induce the large informal sectors observed in developing countries. The main point of this paper is to argue that human capital considerations are the main driving force behind the large informal sectors in the developing world. People with low human capital, especially as it pertains to their ability as employees, opt to become small poor entrepreneurs and populate the informal sector.

In this paper we describe the size distribution of firms and informality in Mexico and in the United States as well as some measures of the distribution of cognitive skills in the population. We also compare the distribution of income across workers and entrepreneurs to better understand their occupational choices. We then build a model in which the size distribution of firms is jointly determined by the distribution of abilities in the labor force and by labor market regulations.

We construct a model that endogenously determines the occupational choices of the population between working as wage-employees or running a business, the size distribution of firms, and the fraction of formal workers in each firm. Several forces are at play in determining occupational choices and the size distribution of firms. (i) As in [Lucas(1978)] agents differ in their entrepreneurial talent and the size distribution of firms depends on the distribution of this type of ability in the population. (ii) We introduce a second source of heterogeneity among agents that is their ability as wage employees. We have in mind that individual differ in their ability to work in teams, understand and respond to instructions, be punctual, presentable and courteous, etc. The basic extension of the [Lucas(1978)] model in this dimension is outlined in [Jovanovic(1994)]. Agents choice of being workers or entrepreneurs depends on the interplay between their talent bundles and the market forces that determine equilibrium wages and profits. (iii) Finally, we assume that tax enforcement is imperfect and that firms are audited with a probability that is increasing in the size of the firm. The idea is that it is inefficient for tax authorities to pay lawyers and accountants to audit and prosecute myriads of poor small business owners that evade taxes (see [Bigio and Zilberman(2011)]).

The model is very successful in matching key moments of the data and we use it to perform several experiments. It is able to match the fraction of entrepreneurs in the population, the size distribution of firms, formality rates across these firms, and the distribution of income across firms and entrepreneurs. In particular, the model is able to reproduce the fact that the distribution of income of wage-employees and entrepreneurs have similar central moments and thicker tails for entrepreneurial income. As in the data, there are many more poor entrepreneurs than wage-employees.

We find that the distribution of skills in the population has a large impact on occupational choices and labor market outcomes. We show that for each agent there is a critical value of her entrepreneurial talent above which she will try her luck as an entrepreneur. This critical value is an increasing function of her abilities as an employee. This implies that the plethora of small businesses that are ubiquitous in the developing world are led by people with low talents as employees. Their choice reveals that they would have done worse working for a larger firm as wage employees than running a small low productivity business. This is consistent with the fact that the poor self employed typically answer in survey questions that they are informal because they could not get a better job. They are entrepreneurs "out of necessity".

We use the model to evaluate several policies that eliminate or reduce the distortions created by the different *de facto* tax treatment of formal and informal firms. We find that changes in taxes or in tax enforcement technologies could increase output by values that are slightly above 1% of output (50% of output if we consider the present value of the 1% stream at an interest rate of 2%).

In the data we find that the distribution of cognitive abilities measured through student achievement tests is very different in Mexico and in the United States. While in the United States about 8% of students do not attain basic numeracy and literacy skills, in Mexico this number is close to 50%. Another difference in the abilities of the labor force is that only one third of the Mexican labor force finished high school while almost 90% of adults in the US did. Student assessment tests also show that abilities are more concentrated around the mean in Mexico than in the United States. The ratio of the standard deviations of the US and Mexico in these tests ranges from 1.14 in reading to 1.27 in science.

We ask the model, which was calibrated to match Mexican data, what would happen if the skills of the population were to increase and if they became more dispersed.

The strongest effect of the different experiments on changing skill distributions is the one we get for increasing people's abilities as employees. An increase in the skills of each agent in the population as wage-employees produces a dramatic selection effect. The entrepreneurs in the left tail of the distribution, about 50% of entrepreneurs, decide to become employees when their working talent increases. This change in occupational choices increases the labor supply and keeps wages constant in spite of the increase in skills. The entrepreneurs on the right tail now demand more labor, and hence formality rates increase by 50%. Output increases by 270%.

Increasing the variance of entrepreneurial talent in the population has a similar effect. Bad entrepreneurs become worse and the good ones better. This induces bad entrepreneurs to become workers and good ones to expand their firms. As a result wages go up by 180%, formality rates increase by 20% and output expands by 190%.

Combining the effect of these two changes in the skill distribution, increasing working abilities and increasing the standard deviation of (log) entrepreneurial talent, is explosive. Output is multiplied by more than 10.

The paper is organized as follows. Section 1 presents some data on the size distribution of firms, informality and the distribution of abilities in Mexico, and compares it with data from the United States and other Latin American countries. In Section 2 we develop a model that endogenously determines the size distribution of firms, the income distribution of workers and entrepreneurs, and the fraction of formal workers in each firm. In Section 3 we describe a solution method and calibrate the model to Mexican data. Finally, in Section 4 we perform some comparative statics exercises to analyze the effects of changes in policies (taxation and auditing) and changes in the underlying distribution of skills.

1 The Distribution of Firms, Informality and Ability

In this section we provide some data on the size distribution of firms, informality and the distribution of abilities in Mexico. We also compare the Mexican data with the US and other Latin American Countries.

1.1 The Size Distribution of Firms and Informality

We start by looking at informality and the size distribution of firms in Mexico. We follow [Busso, Fazio, and Levy(2012)] in defining informality with reference to the observance of a particular regulation. Formal workers are those covered by regulations on salaried labor (enrolled with the Mexican Social Security Institute—IMSS). As labor regulations are imperfectly enforced, salaried workers are hired formally or informally (without paying social security taxes). Non-salaried workers (workers in a cooperative or family enterprise who share benefits, door-to-door sales persons, self-employed workers, . . .) are thus informal, but not illegal, since they do not have to pay social security taxes.

Table 1 shows the size distribution of firms and their employment shares together with the fraction of informal workers for each firm size. There are about 43 million workers in Mexico of which 33 million are urban workers in the private sector. The economic census covers only 60% of these workers[1]. 45% of employees working in establishments with a fixed address (covered in the census) work in places with less than 5 employees. The fraction of workers in mobile production units (not covered by the census) of less than 5 employees is of 78%. The fraction of all private urban employment in production units of less than 5 workers adds up to 58%. Almost all workers in these small production units are informal.

Table 1: Size Distribution of Firms by Employment and Informality

	Number of Workers	Share of employment	Share of informal
Private Urban Employment captured in Census	**19,629,890**		**55.5**
0-5 employees	8,770,687	45%	93.2
6-10 employees	1,714,678	9%	57.2
11-50 employees	3,791,630	19%	28.0
50+ employees	5,352,895	27%	12.8
Private Urban Employment not Captured in Census	**13,223,008**		**86.8**
Self Employees	4,073,747	31%	99.8
2-5 employees	6,228,533	47%	96.6
6+ employees	2,920,728	22%	48.0
Public Sector Employment not Captured in Census	**4,645,104**		0
Rural Employment not Captured in Census	**5,638,429**		95
Total Employment	**43,136,431**		**64.3**

Source: [Busso, Fazio, and Levy(2012)] based on Mexico's 2008 Economic Census and ENOE (National Survey of Employment and Occupation)

Table 2 compares the size distribution of firms in Mexico to that of the United States. It shows that the distribution of employment in the two countries is very different. In Mexico most workers, 58%, are employed in small firms and in the United States the opposite is true, only 4%. Excluding workers that report that they are self-employed, the fraction of private urban employment in small firms with less than 5 employees is an order of magnitude smaller in the United States than in Mexico. On the other hand, 58% of the US labor force

[1] As we will later work with data from Mexico's Economic Census we report the size distribution of firms that are and that are not captured in the census. The economic census excludes economic activity that takes place in mobile units (street vendors and the like), in rural areas, and in government offices.

works in establishments with more than 100 workers and in Mexico only 16% of employees work in establishments of more than 50 workers. The nature of the self employed is also very different in the two countries. In Mexico almost all the self-employed are informal while in the United States a third of the self employed are incorporated. The incorporated self-employed in the US have a large participation of independent professionals (47.6% have a college or an advanced degree). The unincorporated are less educated (41% did not attend college).

Table 2: Size Distribution of Firms by Employment in Mexico and in the United States.

Mexico			United States		
Self Emp	4,073,747	12%	Self Emp	15,148,000	11%
0-5	14,999,220	46%	1-4	6,086,291	4%
6-10	4,635,406	14%	5-9	6,878,051	5%
11-50	3,791,630	12%	10-19	8,497,391	6%
50+	5,352,895	16%	20-99	20,684,691	15%
			100+	78,757,127	58%
Total	32,852,898			136,051,551	

Note: The Mexican data adds all urban private employment in Table 1. US Data is from the Statistics about Business Size (including Small Business), U.S. Census Bureau for paid employees in firms. Self Employment data is for non-agricultural incorporated and unincorporated self employed from the BLS Current Population Survey. The incorporated self employed are 5.591.000. All data is for the year 2007

We conclude from the data in Table 1 and Table 2 that the Mexican economy has a large share of employment in small firms relative to the United States and that most workers in those small firms in Mexico are informal.

1.2 The Size Distribution of Firms, Productivity and Wages.

Table 3, reproduced from [Busso, Fazio, and Levy(2012)], shows that these small establishments where most Mexican employees work (informally) are very unproductive. The table is based on 2008 census data for the manufacturing, retail and wholesale trade, and service sectors. As it was the case for the private urban economy as a whole, most employment in these sectors is in small firms, 38% of workers are in firms of less than 5 employees. This number is somewhat smaller than the aggregate census number of 45% and the economy wide number of 58%. These workers produce only 10% of the value added with 13% of the capital and have an output per worker that is 27% of the average output per worker for these sectors. Workers in large establishments, with more than 50 workers, produce 72% of the value added with 72% of the capital. Their output per worker almost doubles the average one.

Table 3: Size Distribution of Firms and Productivity: Mexico 2008.

	Establishments	Workers	Capital	Value Added	Relative Productivity
0-5 employees	89.7	37.8	13.2	10.3	0.27
6-10 employees	5.8	8.8	4.5	4.6	0.52
11-50 employees	3.6	14.9	10.2	12.5	0.84
+50 employees	0.9	38.5	72.1	72.5	1.88

Source: [Busso, Fazio, and Levy(2012)] based on 2008 Mexican Economic Census. Includes only Manufacturing, Retail and Wholesale Trade, and Services. The last column is the ration of the 5^{th} to the 3^{rd} column and measure the value added per worker of each class of firms relative to the aggregate value added per worker for all firms

Table 4 compares the share of the wage bill and the wages paid by firms of different sizes in the US and in Mexico. The table is based on economic census data for 2007 in the United States and for 2008 in Mexico. We observe that the share of employment and of the wage bill as a function of firm size in Mexico is U-shaped and wages (relative to the average wage) are an increasing function of firm size. Workers in large firms (50+) earn, on average, more than twice the wage of workers in small firms. In the United States, instead, employment shares and wage bill shares are an increasing function of firm size. Wages in the United States are much less dispersed. Average wages in large firms are 40% higher than in medium firms (6-10 employees). The high relative wages of the self-employed in the US are probably due to the large share of high skilled individuals among the self employed, especially the incorporated ones.

Figure 1 reports the distribution of wages and profits in Mexico's 2004 National Survey of Urban Employment (ENEU). This survey contains a question that asks about the nature of the income that respondents derive from their principal occupation. We classify as entrepreneurs persons whose primary income can be thought of as a residual claim on an economic activity[2] Wage employees are people that are paid a fixed wage, an hourly wage, a commission, tips, or by piece work. The income reported in the horizontal axis of figure 1 is normalized by the average income of wage employees and reported in log scale. The figure shows that both distributions have a similar mean, median and mode and they are also skewed towards the left. A second interesting aspect of the comparison between these two distributions is the fatter tails of the profit distribution. In particular, it is interesting to observe that a large number of entrepreneurs is at the bottom of the income distribution. [Allub and Erosa(2012)] report very similar findings for Brazil and show that most of the

[2] These are people that responded in the ENEU that income from their principal occupation is from benefits (ganancias), from what the family produces or sells, or who are paid in merchandize (and we assume trade for profit)

Table 4: Size Distribution of Firms and Wages: Mexico and the United States

	Establishments	Workers	Wage Bill	Relative Wage
Mexico				
0-5 employees	89.7	37.8	23.3	0.62
6-10 employees	5.8	8.8	7.2	0.82
11-50 employees	3.6	14.9	14.1	0.95
+50 employees	0.9	38.5	55.4	1.44
United States				
0-4 employees	48	5	5	0.92
5-9 employees	14	6	4	0.77
10-19 employees	9	7	6	0.81
20-99 employees	9	17	15	0.88
+100 employees	20	65	70	1.08

Source: Mexican data is based on 2008 Economic Census. US data is from Table 2b of the US 2007 Economic Census (http://www.census.gov/econ/smallbus.html). Relative wages are the ratio of the share of the total wage bill over the share of employment

entrepreneurs on the left tail of the distribution are self-employed.

Table 5 reports average wages per quartile in the ENEU dataset normalizing average wages in the first quartile to one. The underlying data is the same as in figure 1.

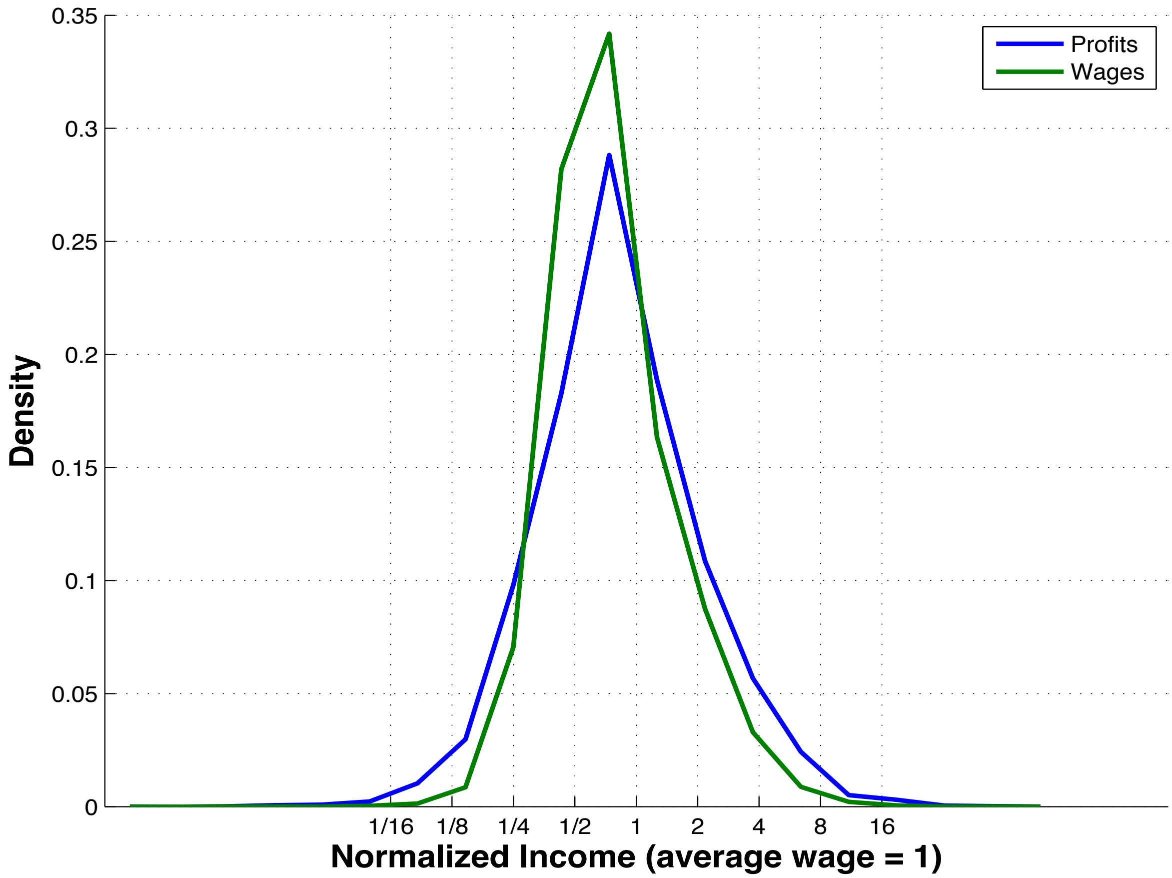
Table 5: Wage and Profit Distribution

Percentile Interval	Average Wage	Average Profit
0-25	1.00	1.00
25-50	1.60	2.08
50-75	2.37	3.50
75-100	6.14	10.65

Note: Wages are normalized to one in the first quartile.
Source: Enucuesta Nacional de Empleo Urbano 2004

We now look at the size distribution of firms creating bins according to their wage bill instead of according to the number of workers using 2008 census data. We order firms according to their total payroll and group them in percentiles. We measure informality as the fraction of required social security contributions to IMSS that firms actually pay. Table 6 presents the previous information in a slightly different way that is readily comparable with

Figure 1: Distribution of Profits and Wage Income in Mexico (2004)



Note: We classify as entrepreneurs persons that responded that the income they receive from their principal occupation are benefits (ganancias), that they consume what the family produces or sells, or that are paid in merchandize from their principal occupation People that are paid a fixed wage, an hourly wage, a commission, tips, or by piece work are classified as employees. Wage and profit income are normalized by the average wage.
Source: Encuesta Nacional de Empleo Urbano 2004.

the predictions of the model in Section 2. The largest 5% of firms by the size of their wage bill, paid 72.5% of wage income in Mexico and these firms paid 85% of their theoretical contributions to social security (IMSS). As most of the wage bill is concentrated on the more formal larger firms, the aggregate measure of informality based on the share of the wage bill that complies with social security contributions is about two thirds. This is in sharp contrast to the share of workers that are informal that is close to 60%.

Summing up, we have learned that (i) most employment in Mexico is in small unproductive firms and that most of the workers in these firms are unproductive. (ii) 58% of Mexican workers are informal, (iii) The distribution of income for workers and entrepreneurs

Table 6: Size Distribution of Firms by Payroll and Informality

Percentile Interval	Share of Payroll	Formality Rate	Share of Employment	Relative Wage
0-25	0.027	0.014	0.082	0.332
25-50	0.044	0.024	0.090	0.489
50-75	0.069	0.059	0.131	0.532
75-90	0.081	0.170	0.131	0.615
90-95	0.054	0.358	0.077	0.702
95-100	0.725	0.853	0.489	1.481
Aggregate		0.657		

Note: Firms are ordered by their total payroll and assigned to bins. Share of Payroll is the fraction of the economy wide payroll paid by firms in each bin. Formality rate reports social security contributions reported in the census as a fraction of those required by law. Relative wage reports the average wage for firms in each bin (Payroll/Employees). Source: Mexico's 2008 Economic Census.

is very similar, but that of entrepreneurs has fatter tails, (iv) workers in the top quartile of the income distribution on average make more than six times the wage of those in the first quartile, (iv) the formality rate measures by the share of the payroll that complies with social security taxes is 66%, (v) the size distribution of firms in the United States is very different with economic activity concentrated in large more productive firms.

1.3 Comparing the distribution of abilities in Mexico and the US

This section presents data on the distribution of abilities in Mexico. For comparison purposes we also include data on advanced countries and on other Latin America countries.

We first present international measures of cognitive skills from [Hanushek and Woessmann(2009a)] and regional measures of cognitive skills for Latin America. Between 1964 and 2003, twelve different international tests of math, science, or reading were administered to a voluntarily participating group of countries. The assessments were designed to identify a common set of expected skills, which were then tested in the local language. It is easier to do this in math and science than in reading, and a majority of the international testing has focused on math and science. [Hanushek and Woessmann(2009a)] construct consistent measures across tests that allow us to compare performance across countries even when they did not each participate in a common assessment[3]. The scale of these assessments is calibrated so that each age group and subject in the tests is normalized to the PISA standard of mean 500 and

[3] The details of this construction as well as the data for all countries is in appendix B of their paper.

individual standard deviation of 100 across OECD countries. A very interesting aspect of the [Hanushek and Woessmann(2009a)] paper is that they provide data on the distribution of student achievement within counties. They calculate the share of students in each country who reach at least basic skills as well as those who reach superior performance levels. They use a test score of at least 400 in their transformed international scale—one standard deviation below the OECD mean—as the threshold for basic literacy and numeracy and a threshold of 600 points for superior performance[4].

Table 7 reports data on cognitive abilities as measured by the test score in international student achievement assessments in seven Latin American countries and an average for selected advanced and East Asian countries. The cognitive skills of the average Latin American student are below what is considered basic literacy in advanced countries. When we look at the share of students that fail to achieve basic skills we observe that about half of the students in Latin America do not attain basic skills in international tests while in advanced countries this fraction of students is 8% and in East Asian countries only 4%. On the other side of the distribution, in Latin America a tiny fraction of students show superior cognitive skills, while in advanced countries almost 9% of students do.

Compare these numbers on the share of students with superior cognitive skills with the 2% of the labor force in management position in the public and private sector[5] or with the 11% of workers in management position in the United States (excluding professional occupations).

The third column in Table 7 reports the average performance of Latin American students in regional tests rescaled to the international metric. The international tests that are designed primarily for developed countries (who support the testing in general) can accurately place student performance near the OECD mean but are thin in questions that would allow discriminating among performance in the tails of the distribution. As a result, the worldwide tests may be unable to distinguish reliably among varying levels of learning in the region of Latin American students. The limitations of worldwide tests in discriminating at the level of Latin American performance leads us to turn to two regional achievement tests specifically designed for the Latin American countries. In 1997, the Latin American Laboratory for the Assessment of Quality in Education Laboratorio Latinoamericano de Evaluacion

[4] The PISA 2003 science test uses the threshold of 400 points as the lowest bound for a basic level of science literacy (Organization for Economic Co-operation and Development (2004), p. 292), and on the math test this corresponds to the middle of the level 1 range (358 to 420 test-score points), which denotes that students can answer questions involving familiar contexts where all relevant information is present and the questions are clearly defined. A score of 600 points is near the threshold of the level 5 range of performance on the PISA 2003 math test, which denotes that students can develop and work with models for complex situations, identifying constraints and specifying assumptions; they can reflect on their answers and can formulate and communicate their interpretations and reasoning.

[5] Funcionarios y directivos de los sectores público, privado y social from INEGI. Encuesta Nacional de Ocupación y Empleo, Third Quarter 2012.

Table 7: International Data on Cognitive Skills.

	cognitive skill Regional	cognitive skill International	basic literacy	top performers
Argentina	395	392	49%	2.7%
Brazil	390	364	34%	1.1%
Chile	413	405	62%	1.3%
Colombia	361	415	64%	0.05%
Mexico	371	400	49%	0.9%
Peru	332	313	18%	0.2%
Uruguay	455	430	62%	4.9%
Average	388	388	48%	1.6%
Advanced Countries		499	92%	9%
East Asia		533	96%	17,3%

Note: a. Cognitive skills for the international tests are from HW (2013). The score is the average test score in math and science, primary through end of secondary school, all years with tests between 1963 and 2003 (scaled to PISA) The OECD average is 500 and the standard deviation is 100. For the regional tests in Latin America the cognitive skills score is a combined measure of LLECE and SERCE performance mapped on the worldwide metric (see H&W 2012). b. Share of students reaching basic literacy in international tests (a score of at least 400 in the Pisa scale). c. Share of top-performing students in international tests (a score of at least 600 in the Pisa scale). d. Selected advanced Countries: Austria, Belgium, Finland, France, Germany, Netherlands, Norway, Portugal, Sweden, Switzerland, United Kingdom, Australia, Canada, New Zealand, United States East Asia: Hong Kong, Japan, Korea, Rep., Singapore, Taiwan.
Source: [Hanushek and Woessmann(2009a)] and [Hanushek and Woessmann(2009b)].

de la Calidad de la Educacion (LLECE) carried out the First International Comparative Study in Language, Mathematics, and Associated Factors in the Third and Fourth Grades of Primary Education (Primer Estudio Internacional Comparativo) specifically designed to test educational achievement in Latin American countries. In 2006, the Latin American bureau of the UNESCO also conducted the Second Regional Comparative and Explanatory Study (Segundo Estudio Regional Comparativo Explicativo, or SERCE). SERCE tested the performance in math and reading of representative samples of students in third and sixth grades. Column 3 in Table 7 reports the average of the country medians of these tests for the students in fourth and sixth grade rescaled to the international metric computed by [Hanushek and Woessmann(2009b)]. We exploit the heterogeneity in cognitive skills across

Latin American countries and data on informality to analyze the relation between student cognitive skills and informality in figure 3.

Figure 2: Distribution of Math and Reading Pisa scores for Mexico and USA (2009)

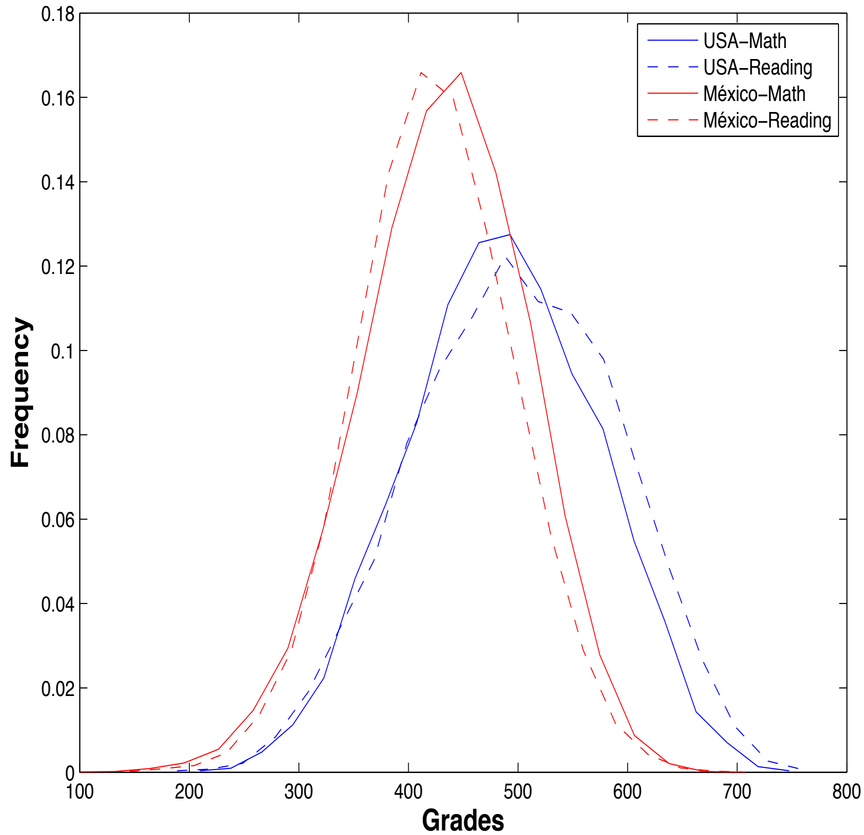


Figure 2 illustrates the statistics in Table 7 with the test-scores of Mexican and US fifteen year old students evaluated in the OECD Programme for International Student Assessment (PISA) in 2009 for math and reading. Observe that the distribution of grades in the two tests is similar within each country. We also see that about one half of Mexican students fall bellow the 400 threshold for basic skills and that only a tiny fraction has scores above 600.

In addition to the differences in student achievement rates there are significant differences in educational attainment levels between Mexico and the United States. While in 2010, 87% of the US population older than 25 years graduated from high school, in Mexico *only a third* of the population of more than 25 years finished high school[6]. For the younger cohorts the numbers look better. In 2010, 70% of the population between 15 and 19 years completed at

[6] Educación media superior

least the basic education (5-15) in Mexico.

In conclusion, only a third of the Mexican population finished high school and those who did attend high school did very poorly in the tests designed to measure cognitive skills when compared to the student population of advanced countries. About half of the students did not attain the basic math and literacy skills of advanced countries and less than 1% of Mexican students attained superior skills.

1.4 Informality and Cognitive Skills

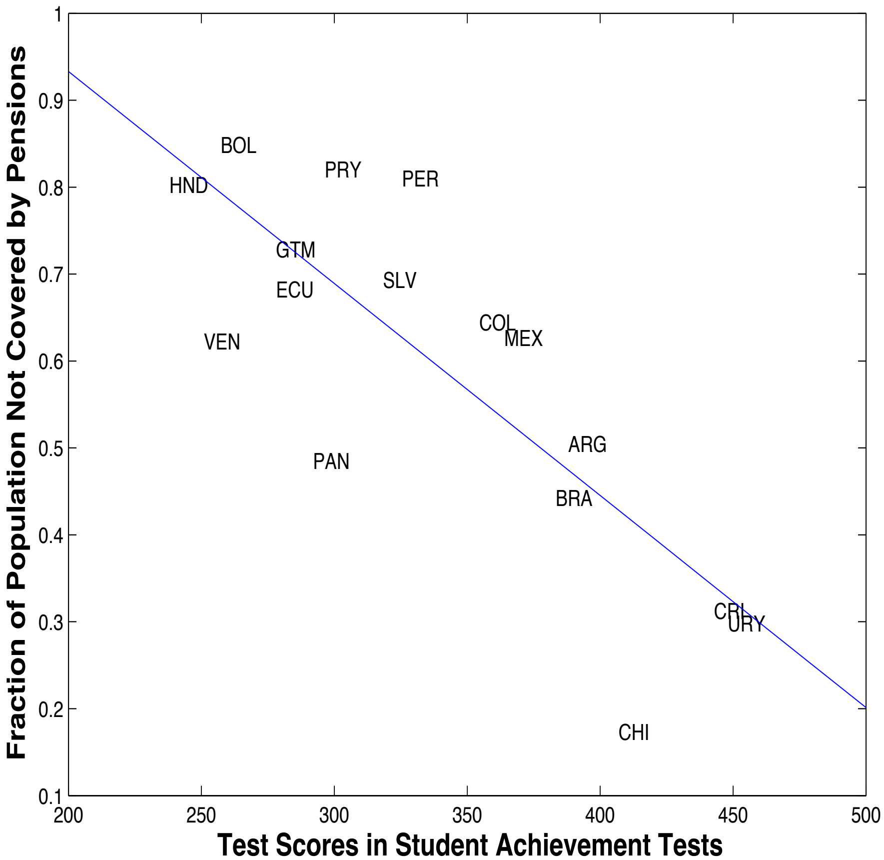
As a rough first approximation of the relation between informality and cognitive skills we plot the fraction of the population not covered by pensions (a proxy for the degree of informality in labor markets) against the cognitive skills measured by the regional test for sixteen Latin American countries in figure 3. It shows that there is a strong negative correlation between cognitive skills and informality (the correlation coefficient is -0.8 and the R^2 of the regression of informality on skills is 0.64).

2 The Model

We now write a model of occupational choice, the size distribution of firms and informality. The model endogenously determines who becomes an entrepreneur or a wage-employee, the size distribution of firms, the income distribution of workers and entrepreneurs, and the fraction of formal workers in each firm. Several key assumptions play an important role in our model. (i) As in [Lucas(1978)] agents differ in their entrepreneurial talent and the size distribution of firms depends on the distribution of this type of ability in the population. (ii) We introduce a second source of heterogeneity among agents that is their ability as wage employees. We have in mind that individuals differ in their ability to work in teams, understand and respond to instructions, be punctual, presentable and courteous, etc. The basic extension of the [Lucas(1978)] model in this dimension is outlined in [Jovanovic(1994)]. (iii) Finally, we assume that tax enforcement is imperfect and that firms are audited with a probability that is increasing in the size of the firm (see [Áureo De Paula and Scheinkman(2011)] and [Bigio and Zilberman(2011)]).

The economy is populated by a continuum of agents with mass 1. Each household is endowed with an ability bundle $z \equiv (z_i^e, z_i^w)$ of entrepreneurial talent, z_i^e, and efficiency units of labor as wage employees, z_i^w. Talents are jointly distributed according with the probability density function $\Phi(z_i^e, z_i^w)$ on the positive real numbers, which we assume to be a bivariate lognormal distribution, $\ln z \sim \mathcal{N}(\lambda_z, \Sigma_z)$ where λ_z and Σ_z are the mean and the variance-

Figure 3: Cognitive Skills and Informality in Latin America



Note: The test scores in student achievement tests are the combined measure of LLECE and SERCE performance in regional tests mapped on the worldwide metric from [Hanushek and Woessmann(2009b)].

covariance matrix of the joint distribution of entrepreneurial abilities and abilities as workers. One possible interpretation of this formulation is to think that agents' abilities as workers and entrepreneurs (in logs) are a linear transformation of the cognitive abilities measured in the student assessment tests reported in table Table 7 plus some random ability shock, so that $\ln z = \mathcal{T}A + \varepsilon$. where $A \in \mathbb{R}^2$, ε is a bivariate normal random variable and $\mathcal{T}$ is the measured cognitive skill in student assessment test.

Agents choose whether to become workers or entrepreneurs. Agents are endowed with a unit of time that can be allocated to work for a wage w per efficiency unit of labor as an employee (i.e. agent i's earnings as an employee are wz_i^w) or to work as an entrepreneur earning profits $\pi(z_i^e, w)$. Agents will self-select comparing their earnings as employees with their profits as entrepreneurs. The set of entrepreneurs $\mathcal{A}$ will be the set of agents that are better-off as entrepreneurs than working as employees.

$$\mathcal{A}(w) = \{(z_i^e, z_i^w) \mid \pi(z_i^e, w) \geq w z_i^w\} \tag{1}$$

Entrepreneurs produce differentiated goods, and their talent bundle z_i also indexes the type of good. Note that, as some agents will not be entrepreneurs, some of the potential differentiated goods will not be produced. The labor market is assumed to be competitive, while the goods market is assumed to be monopolistically competitive.

An entrepreneur with talent bundle z_i produces with the linear technology $Y_i = z_i^e L_i$, where $L_i = \int_0^{l_i} z_{ij}^w dj$ are the efficiency units of labor employed by firm i, with z_{ij}^w being the ability of a worker of type j hired in firm i. As each worker supplies a unit of time, effective labor is given by the sum of the working abilities hired by each entrepreneur. Observe that worker types are perfect substitutes, so the number of employees, l_i, (bodies) hired by each firm i is undetermined.

2.1 Households

Households face the decision problem of how much to consume of each good that is produced and of allocating their unit of working time to work as employees or as entrepreneurs. Let c_{ij} be agent j's consumption of good i -i.e. the good produced by the entrepreneur whose skill bundle is z_i. Agent j's optimization problem is:

$$\max_{\{c_{ij}\}_{\left(z_i^e, z_i^w\right)\in\mathcal{A}}} \left(\int_{\mathcal{A}} c_{ij}^{\frac{\sigma-1}{\sigma}} d\Phi\left(z_i^e, z_i^w\right)\right)^{\frac{\sigma}{\sigma-1}}$$

$$\text{subject to } \int_{\mathcal{A}} \frac{P_i}{P} c_{ij} d\Phi\left(z_i^e, z_i^w\right) \leq \max\left\{w z_j^w, \pi\left(z_j^e, w\right)\right\} + T.$$

where $P = \left(\int_{\mathcal{A}} P_i^{1-\sigma} d\Phi\left(z_i^e, z_i^w\right)\right)^{\frac{1}{1-\sigma}}$ is the price index, $\pi\left(z_j^e, w\right)$ are the expected profits as an entrepreneur[7], and T is a lump-sum government transfer.

Cost minimization implies the demand for good i from consumer j is

$$c_{ij} = \left(\frac{P_i}{P}\right)^{-\sigma} \left\{\max\left[w z_j^w, \pi\left(z_j^e, w\right)\right] + T\right\}$$

[7] We implicitly assume that entrepreneurs receive a lump sum transfer equal to the difference between actual and realized profits. Alternatively, we could assume a complete set of financial markets.

Aggregate demand for each variety is

$$Y_i = \left(\frac{P_i}{P}\right)^{-\sigma} Y, \tag{2}$$

where $Y = \left(\int_{\mathcal{A}} Y_i^{\frac{\sigma-1}{\sigma}} d\Phi\left(z_i^e, z_i^w\right)\right)^{\frac{\sigma}{\sigma-1}}$.

2.2 Informality, labor costs and firm size

In this section we derive the optimal degree of informality as a function of firm size.

Government levies a payroll tax τ, and the revenue is given back to the households as a symmetric lump sum transfer T. Tax collection suffers from imperfect enforcement, so firms can avoid tax payment by choosing to operate in the informal sector.

An *Enforcement Technology* is an auditing function, denoted by $p(L)$, and a penalty function, denoted by $\gamma(1-\mu)$.

Firms are audited with a probability p that is an increasing function of the size of the firm, measured by its effective labor. We assume $p(L)$ is the cumulative distribution function of a lognormal distribution with parameters λ_p and σ_p. We refer to $p(L)$ as the auditing function.

The penalty function is an increasing function of the rate of informality, defined as the unpaid fraction of the tax bill. We assume $\gamma(1-\mu_i)$ has the properties $\gamma(1-\mu_i) \geq 1$, $\gamma' > 0$, and $\gamma'' > 0$.

If a firm is not audited, it will pay taxes only on its formal payroll i.e. its profits (expressed in units of aggregate consumption) are

$$\text{Benefits if not audited} = \frac{P_i}{P} Y_i - w\left[1-\mu_i+\mu_i(1+\tau)\right] L_i$$

Labor costs are the product of the quantity of effective labor multiplied by a weighted average of the wage rate w and the wage rate cum tax $w(1+\tau)$, with weights equal to the rate of informality, $1-\mu_i$, and to the rate of formality, μ_i.

An audited firm pays all labor taxes it owes, $(1-\mu_i)\tau w L_i$, multiplied by the penalty function over its tax liability, $\gamma(1-\mu_i)$. Profits for audited firms are

$$\text{Benefits if audited} = \frac{P_i}{P} Y_i - \left[(1-\mu_i)\left(1+\gamma(1-\mu_i)\tau\right)+\mu_i(1+\tau)\right] w L_i$$

Firms choose de fraction of formal workers μ_i and the level of employment L_i to maximize expected profits:

$$\pi_i(z_i^e, w) = \max_{L_i,\mu_i} (1 - p(L_i)) \left[\frac{P_i}{P} Y_i - (1 + \tau_L \mu_i) w L_i\right] + p(L_i) \left[\frac{P_i}{P} Y_i - [1 + (\mu_i + (1 - \mu_i) \gamma (1 - \mu_i)) \tau_L] w L_i\right]$$

Using (2), this can be rewritten as:

$$\pi_i(z_i^e, Y, w) = \max_{L_i,\mu_i} \pi_i(L_i, \mu_i \mid z_i^e, Y, w)$$

where

$$\pi_i(L_i, \mu_i \mid z_i^e, Y, w) = Y^{\frac{1}{\sigma}} (z_i^e L_i)^{1-\frac{1}{\sigma}} - (1 + \tau \mu_i) w L_i - p(L_i)(1 - \mu_i) \tau w L_i \gamma (1 - \mu_i) \quad (3)$$

are the expected profits as a function of L_i, μ_i and the exogenous (from the point of view of the firm) variables z_i^e, Y, w.

The FOC with respect to μ_i can be written as:

$$1 = p(L_i) \gamma \left[1 + \frac{(1 - \mu_i) \gamma'}{\gamma}\right] \quad (4)$$

The FOC (4) imply that the fraction of formal workers within a firm is a function of the probability of detection and of the shape of the penalty function, while it is independent of taxes and wages. The latter matter for informality only through their effect on firm size.

If we assume that $\gamma = \frac{b}{a+\mu_i}$ then the firm's optimal choice of μ_i and γ as a function of firm size are:

$$\mu_i(L_i) = \begin{cases} 0 & \text{if } p(L_i) \le \frac{a^2}{(1+a)b} \\ \sqrt{b(1+a)p(L_i)} - a & \text{if } \frac{a^2}{(1+a)b} \le p(L_i) \le \frac{1+a}{b} \\ 1 & \text{if } \frac{1+a}{b} \le p(L_i) \end{cases} . \quad (5)$$

$$\gamma(L_i) = \begin{cases} \frac{b}{a} & \text{if } p(L_i) \le \frac{a^2}{(1+a)b} \\ \frac{b}{\sqrt{(1+a)bp(L_i)}} & \text{if } \frac{a^2}{(1+a)b} \le p(L_i) \le \frac{1+a}{b} \\ \frac{b}{1+a} & \text{if } \frac{1+a}{b} \le p(L_i) \end{cases} \quad (6)$$

Using the optimal degree of informality we can use the first order conditions with respect to labor to obtain the optimal size of firms. The FOC are

$$\left(1-\frac{1}{\sigma}\right) Y^{\frac{1}{\sigma}}\left(z_i^e\right)^{1-\frac{1}{\sigma}} L_i^{-\frac{1}{\sigma}}=\left(1+\tau \mu_i(L_i)\right) w+\left(1-\mu_i(L_i)\right) \tau w \gamma\left(1-\mu_i(L_i)\right)\left(p\left(L_i\right)+p^{\prime}\left(L_i\right) L_i\right) \tag{7}$$

for all i. Solving for L_i yields a labor demand function $L^*\left(z_i^e, Y, w\right)$.

2.3 Equilibrium

An *equilibrium* for this economy is a set $\mathcal{A} \subset \mathbb{R}_+^2$ and a pair (w, Y) such that

$$\int_{\mathcal{A}} L^*\left(z_i^e, Y, w\right) d\Phi\left(z_i^e, z_i^w\right) = \int_{\sim\mathcal{A}} z_i^w d\Phi\left(z_i^e, z_i^w\right) \tag{8}$$

$$\mathcal{A}(w, Y) = \left\{\left(z_i^e, z_i^w\right) \mid \pi\left(z_i^e, Y, w\right) \geq w z_i^w\right\} \tag{9}$$

In an equilibrium the labor market clears and each agent optimally selects her occupation. The left hand side of equation (8) is the labor demand that solves the firms optimization problem added over all agents that decide to be entrepreneurs. The right hand side of equation (8) is the labor supply—i.e. the sum of the working ability of all those who choose to be workers. Equation (9) defines the set of entrepreneurs on the left hand side of equation (8) and the set of workers, $\sim\mathcal{A}$, in the labor supply on the right hand side.

The following propositions show some useful properties of the equilibrium. We will show that (i) larger firms hire more formal workers, (ii) firm size is an increasing function of the ability of its entrepreneur, z_i^e, and (iii) the cut-off entrepreneurial ability $\bar{z}_i^e$ at which an agent becomes an entrepreneur is an increasing function of her ability as a worker z_i^w.

Proposition 1. The optimal degree of informality is a weakly increasing function of firm size which depends only on the enforcement technology. There exist thresholds $\underline{L}$ and $\bar{L}$ such that:

(i) if $L_i \leq \underline{L}$ firms are informal.

(ii) if $\underline{L} < L_i < \bar{L}_i$ firms are semi formal with $0 < \mu_i\left(L_i\right) < 1$ and $\mu_i\left(L_i\right)$ increasing in L_i.

(iii) if $\bar{L} \leq L_i$ firms are formal.

Proof. From equation (5) the thresholds are $\underline{L} = p^{-1}\left(\frac{a^2}{(1+a)b}\right)$ and $\bar{L}_i = p^{-1}\left(\frac{1+a}{b}\right)$, where p^{-1} is the inverse of the auditing function. Applying the implicit function theorem to equation (4) for $0 < \mu < 1$ yields $\frac{\partial \mu}{\partial L_i} = -\frac{-p'(L_i)[\gamma+(1-\mu)\gamma']}{p(L_i)[2\gamma'+(1-\mu)\gamma'']} > 0$. ■

The following lemma is useful to show that firms size is increasing in entrepreneurial ability.

Lemma 1. Let $f(x)$ and $g(x)$ be differentiable functions such that $g'(x) > f'(x)$ for all x, and let $x_f = \arg\max_x f(x)$ and $x_g = \arg\max_x g(x)$. Then, $x_g > x_f$.

Proof. First, we prove that $g(x_f) \geq g(x)\, \forall x < x_f$. For any $x < x_f$,

$$\begin{aligned} g(x_f) - g(x) &= \int_x^{x_f} g'(s)ds > \int_x^{x_f} f'(s)ds = f(x_f) - f(x) \geq 0 \\ &\Rightarrow g(x_f) > g(x) \end{aligned}$$

This means that $x_g \geq x_f$. From $g'(x_f) > f'(x_f) = 0$, we conclude that $x_g > x_f$. ■

Proposition 2. Firm size, $L^*(z_i^e, Y, w)$, is increasing in entrepreneurial ability, z_i^e .

Proof. From equation (3), $\frac{\partial \pi_i\left(L_i, \mu(L_i) | z_i^e, Y, w\right)}{\partial L_i}$ is increasing in z_i^e. This means that, if we take two entrepreneurial skills z_1^e, z_2^e such that $z_2^e > z_1^e$, we have

$$\frac{\partial \pi_i (L_i, \mu(L_i) \mid z_1^e, Y, w)}{\partial L_i} < \frac{\partial \pi_i (L_i, \mu(L_i) \mid z_2^e, Y, w)}{\partial L_i} \text{ for all} i \tag{10}$$

Then, $L^*(z_2^e, Y, w) > L^*(z_1^e, Y, w)$ follows directly from Lemma 1. ■

Proposition 2 states that firms size is increasing in the entrepreneurial talent of its leader. The proposition takes into account that larger firms pay more taxes through lower informality rates (see Proposition 1). Although it is a partial equilibrium proposition, as the aggregate affect on output and wages of changing the talent of one infinitesimal agent is negligible, it should be valid as a property of the general equilibrium. In a very similar model to this one [Hopenhayn(2012)] shows that in the absence of rank reversals—more productive firms being smaller than less productive firms—distortions that re-allocate resources among heterogenous firms do not have a large impact on aggregate output or productivity. Proposition 2 shows that more productive firms hire more effective labor and, hence, the distortion introduced by informality does not generate any rank reversals. In Section 3 we indeed find small output effect from informality.

Proposition 3. The set $\mathcal{A}$ can be written as $\mathcal{A} = \{(z_i^e, z_i^w) \mid z_i^e \geq \bar{z}_i^e(z_i^w)\}$ where $\bar{z}_i^e(z_i^w)$ is an increasing function of z_i^w.

Proof. The function $\bar{z}_i^e(z_i^w)$ is such that $wz_i^w = \pi(\bar{z}_i^e(z_i^w), Y, w)$. Given that $\pi(z_i^e, Y, w)$ is increasing in z_i^e, the proof of the proposition is straightforward. ■

Proposition 3 states that the critical level of entrepreneurial talent above which agents decide to become entrepreneurs is an increasing function of their talent as employees. This allows for equilibria in which there are entrepreneurs who are in the lower end of the distribution of entrepreneurial talent in the population, but whose entrepreneurial talent is high relative to their talent as workers.

3 Solution Method and Calibration

This section presents the solution method and the calibration method for the parameters.

We solve the model with a discrete approximation. We use 40 values for skill as an employee - i.e. $z_i^w \in \{z_1^w, z_2^w, ..., z_{40}^w\}$, where $z_1^w < z_2^w < ... < z_{40}^w$. For entrepreneurial ability we use 200 values for skill as an entrepreneur - i.e. $z_i^e \in \{z_1^e, z_2^e, ..., z_{200}^e\}$, where $z_1^e < z_2^e < ... < z_{200}^e$. Thus, there are $8,000$ pairs (z_i^e, z_i^w) in our grid. We use a step wise approximation with 40 steps for the auditing function $p(L_i)$. This approximation simplifies equation (7) as it allows us to ignore the term involving $p'(L_i)$ within each step.

The parameters we have to calibrate are:

Table 8: Parameters to Calibrate

τ	labor tax
σ	substitution between varieties
a, b	penalty function
λ_e, λ_w, σ_e, σ_w, ρ_{ew}	joint distribution of skills
λ_p, σ_p	auditing function

The parameters τ, σ and b are set exogenously. We set $\sigma = 3$ as in [Hsieh and Klenow(2009)]. As Mexican legislation stipulates that the penalty for tax evasion of social security contributions is 1.5 times the unpaid tax liability we set $b = 1.5(1 + a)$ and choose a value for a that makes the penalty stay close to 1.5 times the owed taxes.

Firms in Mexico are legally required to pay a $t_L = 32\%$ tax on their payroll. We assume that, for each peso payed by the firm as payroll tax, the employee receives β^{CSI} in benefits (CSI stands for contributory social insurance). However, we do not set $\tau = 0.32$ because we also want to take into account that a further distortion is introduced by the fact that informal workers also receive social insurance benefits (NCSI, non-contributory social insurance). We assume that for each peso spent by the government in NCSI the worker receives β^{NCSI} in benefits. For an employee to be indifferent between being a formal worker receiving a wage w^F (plus CSI benefits) and being an informal worker receiving a wage w^I (plus an NCSI lump-sum transfer of T^{NCSI}), the following arbitrage condition must be satisfied:

$$\begin{aligned} w^F(1 + \beta^{CSI} t_L) &= w^I + \beta^{NCSI} T^{NCSI} \\ w^F &= \frac{w^I + \beta^{NCSI} T^{NCSI}}{1 + \beta^{CSI} t_L} \end{aligned} \tag{11}$$

In order to fit this transfer into the model while keeping the framework developed in the previous section, we would need to express $\beta^{NCSI}T^{NCSI}$ as a fraction of the informal wage w^I. Although the ratio $\frac{w^I}{\beta^{NCSI}T^{NCSI}}$ is not the same for all workers (because w^I depends on the working ability of each individual), we will calculate the implicit tax $\tilde{t}_L$ in (11) for the case of an employee earning the minimum wage established by law, and then apply this same value for all employees (not only the ones earning the minimum wage). Following Levy (2008, 2012), we set $\beta^{CSI} = 0.3$, $\beta^{NCSI} = 0.85$, $T^{NCSI} = 5,652$ pesos per year in 2008. The minimum wage for 2008 was 13,884 pesos per year[8]. Therefore, from (11) we get:

$$w^F = 1.19w^I$$

so $\tilde{t}_L = 0.19$. Then, the value of τ used for our calibration is $\tau = (1 + t_L)(1 + \tilde{t}_L) - 1 = 0.57$.

The rest of the parameters are calibrated by simulating the model and matching moments from the data. λ_e is normalized to $\lambda_e = 0$. The moments we want to match are:

1. The fraction of entrepreneurs in the population. From the International Labor Statistics database, we get that a 26.3% of the economically active population is either an employer or an own-account worker[9], while the rest are employees (this data corresponds to 2008).

2. The wage distribution of employees. Using the 2004 National Survey of Urban Employment (ENEU) as described in Section I, we separate employees in quartiles (each quartile comprises 25% of the workers) and compute the average wage in each of these quartiles.

3. The distribution of payroll among firms. Using the Economic Census, we separate firms into bins according to their payroll; the bins we use are the percentile intervals: 0-25,25-50,50-75,75-90,90-95,95-100. For example, the first bin, 0-25, contains the 25% of firms which have the smallest payroll, and the last bin, 95-100, contains the 5% of firms with the largest payroll. For each of these bins we calculate the ratio between the total payroll of the bin over the total payroll of all firms in the Census to get the size distribution of firms.

4. Size distribution of informality and aggregate informality. We classify firms in bins according to payroll as explained in the previous item (we use the same bins as before).

[8] We consider 22 working days per month.

[9] We exclude the categories "fishing", "mining and quarrying", and "agriculture, hunting and forestry" as the Economic Census only includes urban firms.

For bin j, we calculate the ratio

$$C_j = \frac{IMSS_j}{Payroll_j}$$

where $IMSS_j$ is the total contributions to Social Security (IMSS, Instituto Mexicano del Seguro Social) of firms in bin j and $Payroll_j$ is the total payroll of those firms[10]. According to the Mexican law, firms must contribute to Social Security at least an 18% of employees' wages. Therefore, we will use:

$$\tilde{\mu}_j = \frac{C_j}{0.18}$$

as an index of the degree of formality of firms in bin j[11].

We also perform this same calculation for the whole population in order to get the aggregate fraction of formal labor.

As we have more moments than parameters, we calibrate these parameters by minimizing the weighted sum of square differences between the moments in the model and the data. We give four times more weight to the fraction of entrepreneurs in the population and to the aggregate informality rate. The calibrated parameters are reported in Table 9.

Table 9: Calibrated Parameters

τ	0.57
σ	3
a	1.11
b	3.17
λ_e	0
λ_w	-1.33
σ_e	1.3
σ_w	1.03
ρ_{ew}	-0.8
λ_p	1.45
σ_p	2.58

The value $a = 1.11$ indicates that the penalty γ ranges from 1.5 (when $\mu = 1$) to 2.85 (when $\mu = 0$). In Mexico the penalty is flat at 1.5, which means that in order to replicate

[10]We adjust the payroll by the factor 12/13 to account for the fact that the 13th month of salary is not subject to contributions to social security (the so-called aguinaldo).

[11]If a firm k is such that $\frac{IMSS_k}{Payroll_k} > 0.18$, we replace $IMSS_k$ for $0.18 Payroll_k$ as a firm cannot have more than a 100% of its workers in the formal sector.

informality rates in Mexico the model calls for a steeper penalty function as the fraction of the informal payroll increases.

The size distribution of skills implied by the calibrated parameters of the joint lognormal distribution of skills as entrepreneurs and workers, $(\lambda_e, \lambda_w, \sigma_e, \sigma_w, \rho_{ew})$ is shown figure 4

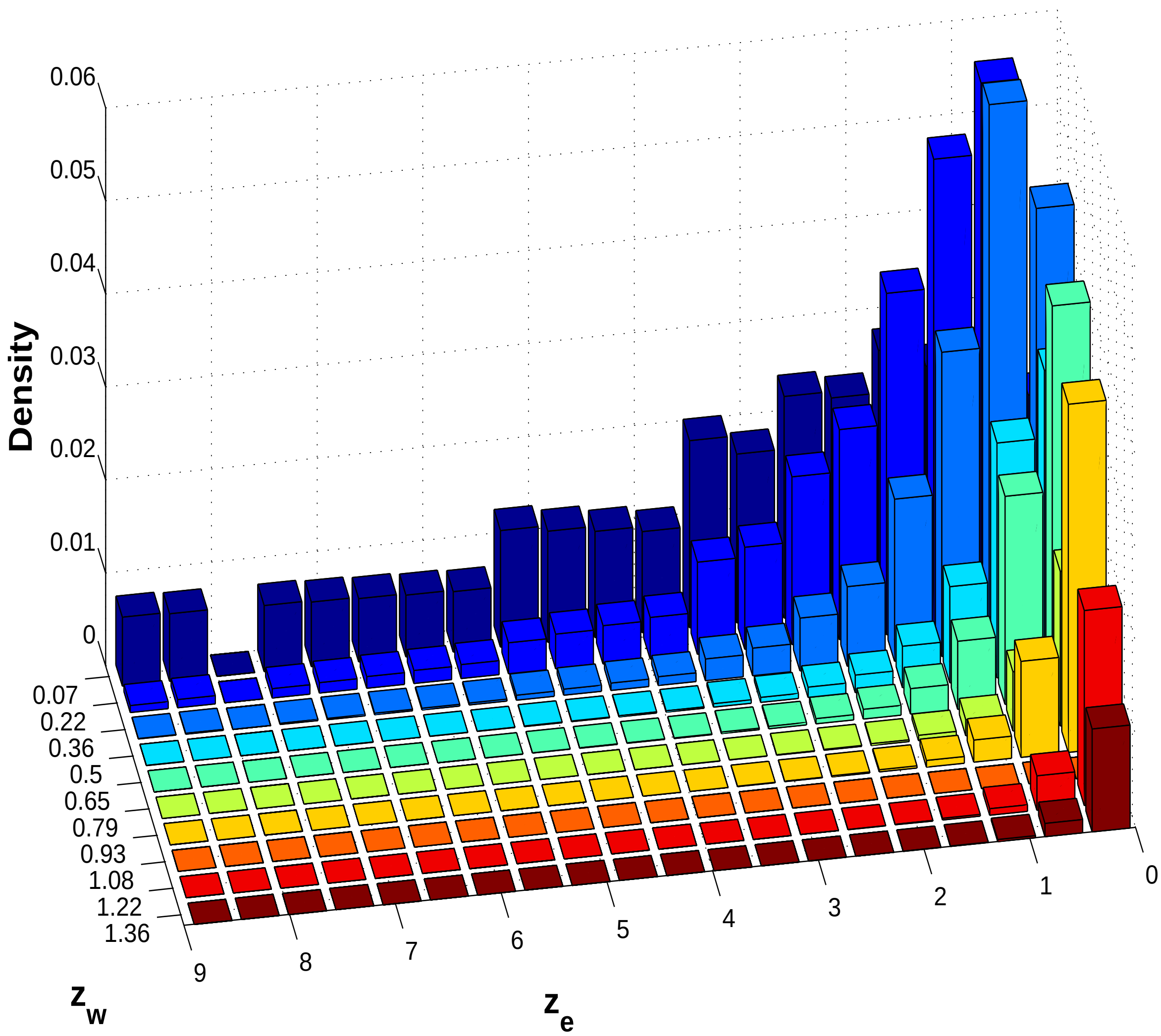
Table 10: Results for Benchmark Calibration

	Percentile Interval	Data	Model
Fraction of Entrepreneurs		0.263	0.231
Workers share of income			0.628
Average wage	0-25	1.00	1.00
(normalized by	25-50	1.60	1.83
first quartile)	50-75	2.37	3.07
	75-100	6.14	6.23
Average profit	0-25	1.00	1.00
(normalized by	25-50	2.08	1.92
first quartile)	50-75	3.50	3.73
	75-100	10.65	26.97
Share of Total Payroll	0- 25	0.027	0.030
by Percentile Interval	25-50	0.044	0.056
	50-75	0.069	0.098
	75-90	0.081	0.133
	90-95	0.054	0.117
	95-100	0.725	0.566
Formality Rate	0-25	0.014	0.000
by Percentile Interval	25-50	0.024	0.000
	50-75	0.059	0.062
	75-90	0.170	0.324
	90-95	0.358	0.586
	95-100	0.853	0.951
	Aggregate	0.657	0.656

Note: Bins for wages are created by ordering employees by wages and comparing the model with ENEU data. Bins for firms are created by ordering them by the size of their payroll and comparing them with census data. The bins for the size distribution of firms and formality rates are the same. Workers' share of income includes labor taxes

Figure 4 shows that in order to fit Mexican data on the fraction of the population that are entrepreneurs, the size distribution of Mexican firms (by payroll) and the size distribution

Figure 4: Calibrated Joint Distribution of Skills as Workers and Entrepreneurs



of informality for production units captured in the CENSUS, and the wage distribution (in ENEU), most individuals abilities are concentrated around zero. As we move away from the origin agents cluster around the edges, which means that they are good either as workers or as entrepreneurs. Our calibration of ρ_{ew} is actually negative and corresponds to a correlation of -0.1 in the levels of z.

Table 10 shows the value of the simulated descriptive statistics which the benchmark calibration for the eight free parameters tries to match to the data. Overall, the model does pretty well in matching the data value of the eighteen target descriptive statistics. It does very well in replicating the fraction of entrepreneurs in the population[12], the distribution of wages, and the overall pattern of the size distribution of firms and of informality. As a bonus we get that the workers' share of income is reasonable. The main deviation is that the model puts too many resources in the 25-95 percentile of firms and not enough in the top 5% as the Mexican data. Formality rates are also higher in the model than in the data for the top 25% of firms. Even though the model is not calibrated to fit the distribution of profits it does pretty well in reproducing the profit of the middle income entrepreneurs relative to the poor ones as the third block of results in Table 10 shows. The relative income of the top quartile entrepreneurs in the model is more than twice the one in the data, but this might be due to the fact that we are capturing *all* high income entrepreneurs and the ENEU survey probably misses them.

The equilibrium in this benchmark calibration is described in figure 5 and in Table 10. The first panel in the figure shows the selection set of entrepreneurs from equation (9). We observe that for each level of worker ability,z_w, entrepreneurs are those with the highest entrepreneurial ability, z_e. We also observe that the critical z_e is increasing in z_w in accordance with Proposition 3. This means that it is possible to have an equilibrium with firms run by entrepreneurs with high entrepreneurial skills relative to their worker ability but low absolute entrepreneurial skills. Panel (b) in the figure shows that firm labor demand is increasing in z_e as in Proposition 2. In panel (c) we see that for small firms (with low z_e) the probability of auditing is low, below the threshold in equation (5) and all workers are informal. As the auditing probability increases firms become semi-formal and the critical z_e above which the auditing probability induces all firms to be formal in equation (5) is never reached. This is probably due to the fact that the formality rate for the top 5% in Mexico does not reach 100%. Panel (d) shows that the value of the marginal product of labor as a function of firm size. For small firms, low values of z_e, the value of the marginal product of labor is low as the marginal cost of labor is low because firms do not pay payroll taxes and auditing probabilities are low. Interestingly, the value of the marginal product of labor is not monotonically increasing. For medium sized firm's increasing their scale of operation implies an increase in the auditing probability that forms part of the marginal cost. The calibration assigns the steepest part of the auditing probability function to these firms. Firms in these range optimally decide to reduce their scale relative to the "pure marginal" cost of labor to increase the chance of not being detected by the tax authority.

The model does well in replicating income distribution despite the fact that it was not calibrated to match these data. In panel (e) in figure 5 we observe the simulated distribution

[12] The fraction of entrepreneurs in the population is from ILO's Laborsta database for Mexico in 2008. The fraction of entrepreneurs in ENEU is 20%, which is easier to hit with the model.

of income for workers and entrepreneurs. This figure corresponds to figure 1 with wages and profits from the data. The model successfully replicates the main features of the data: both distributions have similar central moments and are skewed to the left, there is a higher mass of entrepreneurs at in the tails, including the lower one. The model has too much weight on the right tail relative to the data (high income entrepreneurs might not be captured in surveys) and too little weight on the lower tail. The calibration and equilibrium do not generate enough poor entrepreneurs.

4 Comparative Statics

In this Section we use the model to analyze the impact of changing some of the exogenous parameters. Taking the model economy calibrated in the Section 3 as our benchmark, we perform two types of exercises: policy experiments and changes in the underlying distribution of skills.

The policy experiments we study are changes in the payroll tax and in the auditing function. We ask the model how the equilibrium with the benchmark parameters would change if (i) there is perfect enforcement ($p = 1$ for all firms), (ii) the non contributory social insurance benefits are eliminated, which corresponds according to equation (11) to setting $\tau = 0.20$, and (iii) to reduce payroll taxes to zero, $\tau = 0$.

The second set of experiments is to ask how labor market outcomes, formality rates, income distribution and output would change if the distribution of skills changes. We interpret this as asking if differences in the skill distribution can explain differences between the characteristics of the size distribution of firms and of informality in advanced and developing countries. The changes in the distribution of skills we consider are shifts in the distribution that translate the whole distribution to the right, mean preserving spreads (in logs), and the interaction between the two. The translations of the distribution we consider are (i) to increase all values of z^e by σ_e, (ii) to increase all the values of z^w by σ_w, and (iii) to increase all values of z^e by σ_e and all values of z^w by σ_w. The mean preserving spreads (in $log(z)$) are (i)to increase σ_e by 30%, (ii) to increase σ_w by 30%, and (iii) to increase σ_e and σ_w by 30%. Finally, (iv) the interaction of the two is to increase all values of z^e and z^e by σ_e and σ_w, respectively and to increase the two standard deviations by 30%.

4.1 Policy Experiments

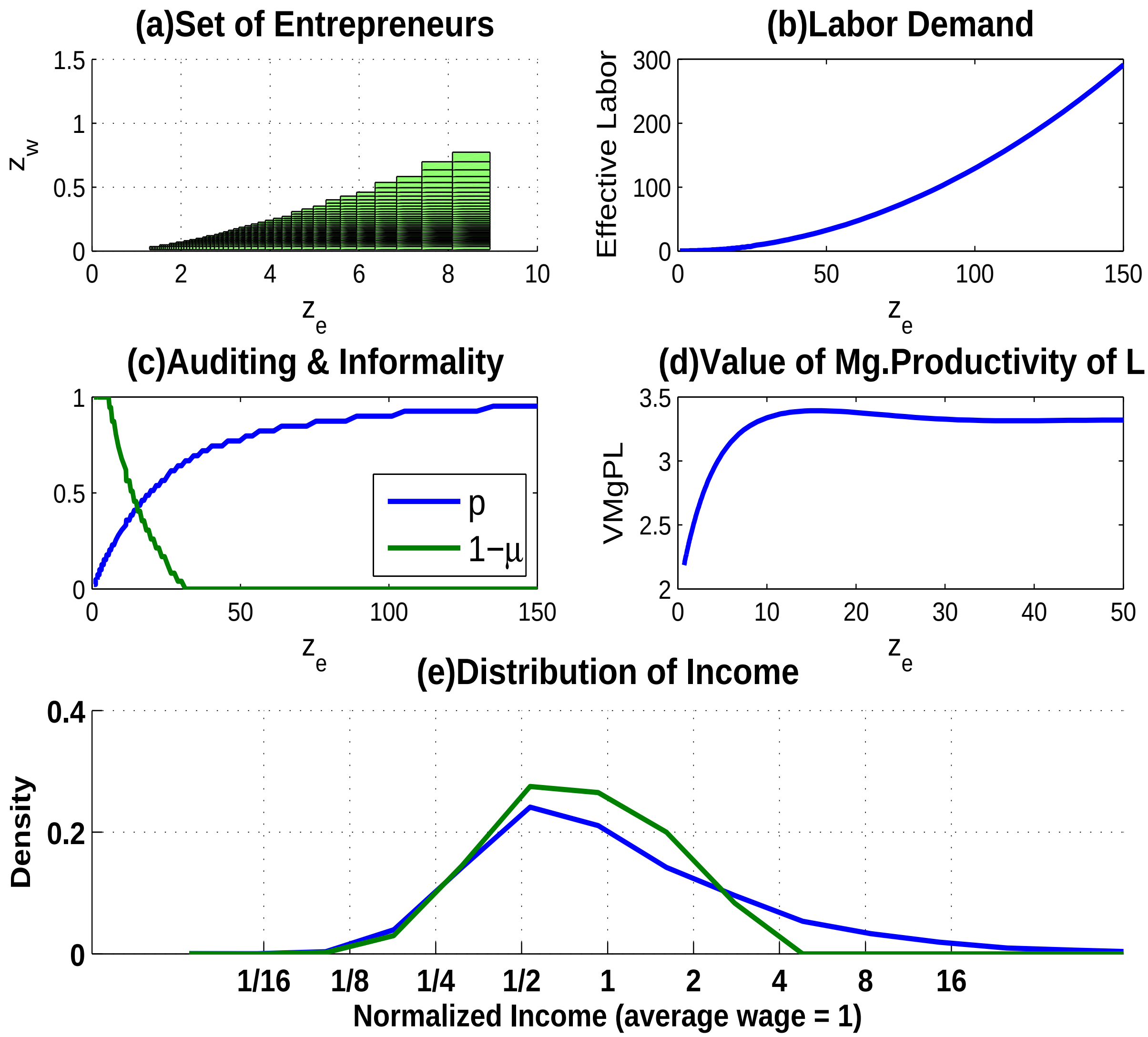
Table 11 and figure 6 report the effect of the three policy experiments in our benchmark calibration. With perfect auditing, the wedge in marginal productivity of labor disappears,

Table 11: Policy Experiments

	Percentile Interval	Benchmark	$p = 1$	$\tau = 20$	$\tau = 0$
Fraction of Entrepreneurs		0.231	0.187	0.185	0.155
Workers share of income		0.628	0.698	0.664	0.69
% change in wages wrt benchmark		0	-0.017	0.286	0.53
% change in output wrt benchmark		0	0.012	0.01	0.014
Average wage (normalized by first quartile)					
	0- 25	1	1	1	1
	25-50	1.83	1.87	1.86	1.95
	50-75	3.07	3.12	3.12	3.36
	75-100	6.23	6.49	6.48	6.99
Share of Total Payroll by Percentile Interval					
	0-25	0.03	0.019	0.024	0.022
	25-50	0.056	0.04	0.044	0.044
	50-75	0.098	0.094	0.096	0.1
	75-90	0.133	0.161	0.154	0.165
	70-95	0.117	0.118	0.114	0.123
	95-100	0.566	0.568	0.567	0.546
Formality Rate by Percentile Interval					
	0-25	0	1	0	1
	25-50	0	1	0	1
	50-75	0.062	1	0.107	1
	75-90	0.324	1	0.41	1
	90-95	0.586	1	0.67	1
	95-100	0.951	1	0.97	1
	Aggregate	0.656	1	0.7	1

Note: Bins for wages are created by ordering employees by wages. Bins for firms are created by ordering them by the size of their payroll. The bins for the size distribution of firms and formality rates are the same. Worker's share of income includes labor taxes. $p = 1$ corresponds to perfect tax enforcement, $\tau = 20$ corresponds to eliminating non-contributory pension benefits, $\tau = 0$ corresponds to eliminating labor taxes.

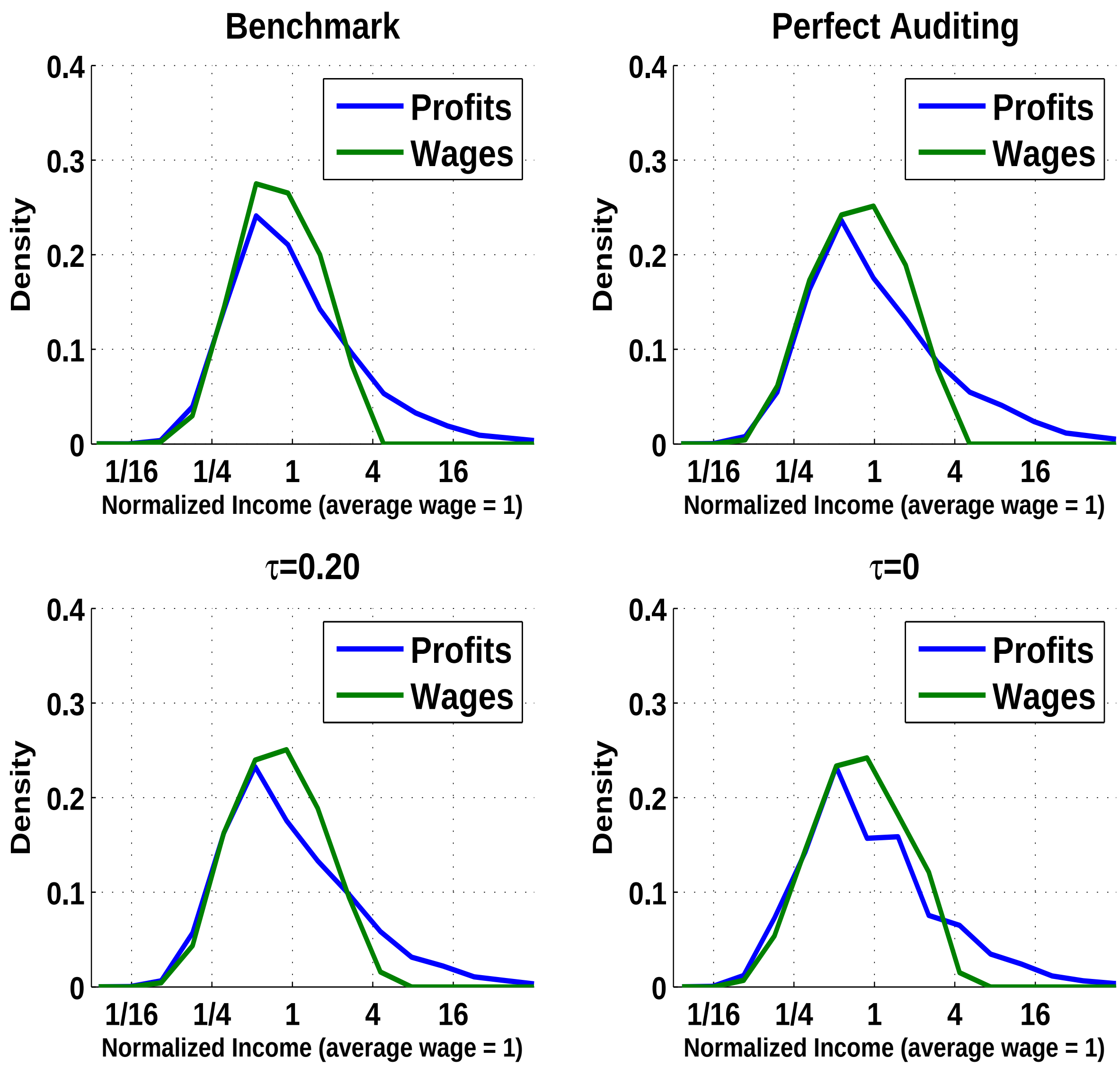
Figure 5: Equilibrium in the Benchmark Calibration



Note: Panels (a)depicts the set of entrepreneurs in equation (9). (b) shows the effective labor demanded for each entrepreneur. (c) auditing probability and fraction of informal payroll by entrepreneurial ability. (d) Value of Marginal product of effective labor by entrepreneurial ability. (e) distribution of income for workers (green) and entrepreneurs (blue)

eliminating the misallocation of resources among heterogeneous firms. The corresponding increase in aggregate product is of 1.2%. As small firms now face a higher labor cost (because they have to comply with payroll taxes in full), some of these entrepreneurs will choose to become workers. This explains the drop in the fraction of entrepreneurs from 23.1% to 18.7%. The corresponding increase in the labor supply induces a fall of 1.7% in wages. The share of very small businesses in the aggregate payroll shrinks as many of these entrepreneurs decide

Figure 6: Policy Experiments



to be wage employees and the share of effective labor employment in medium sized businesses (percentile interval 75-90) increases.

Reducing the tax rate to $\tau = 0.20$ also has the effect of reducing the misallocation of resources among firms because the wedge in marginal productivity of labor diminishes. This yields a 1% increase in output. Once again, entrepreneurs facing higher labor costs choose to become workers, leading to a drop in the fraction of entrepreneurs in the population. Lower taxes however, induce firms to demand more labor and, as this effect dominates over the one of the expanded labor supply, real wages increase by almost 29%. Lower taxes lead to an overall increase in formality rates. Notice however, that this is not because the lower taxes reduce the incentives to evade taxation. Proposition 1 shows that formality increases because the increase in firm size due to the lower marginal cost of labor increase the auditing probability. The last case is qualitatively analogous with quantitatively stronger results.

Figure 6 shows that none of these policy experiments has a significant effect on the distribution of income among workers and entrepreneurs.

4.2 Changes in the Skill Distribution

We first consider the impact of shifting the distribution of skills by adding a standard deviation to the talent bundles z. We do this by first increasing entrepreneurial talent, then increasing working talent and finally increasing both. This exercise is motivated by the wide distance in cognitive abilities in the Mexican and in the US population. The results are reported in Table 12 and in figure 7. The benchmark case is always reported for comparison purposes.

The first exercise increases only entrepreneurial talent, keeping the abilities of agents as workers constant. This experiment increases the fraction of the population who choose to be entrepreneurs from 23% to 30%, reducing the labor supply. At the same time, each firm's labor demand shifts with the parameter z_e. The combined effect is an increase in wages of 12%. The size distribution of firms (associating size to payroll) changes, with smaller firms hiring more effective labor. The increase in the number of small businesses increases the rates of informality as many people that would have been workers now become small entrepreneurs. There are no major changes in the distribution of income of either workers or entrepreneurs as can be observed in Table 12 and figure 7. Output increases by 7.5%.

The second exercise increases only working talent, keeping entrepreneurial talent constant. The effect of this experiment on the income distribution of workers and entrepreneurs in figure 7 is dramatic. Wage income compresses around the mean and entrepreneurial income shifts to the right. The average wage of workers in the top quartile relative to those in the bottom quartile falls from 6.2 to 1.6. As agents' talents as workers are so much better, many more decide to become workers and the left tail of entrepreneurs disappears. Proposition 3 is at work here. Better worker ability kills firms with bad entrepreneurs. This is reflected in the drop in the share of entrepreneurs in the population from 23% to only 12%. The large increase in the labor supply results in the paradoxical situation in which wages fall in spite of the increase in the talent of workers. As firms are much larger due to the strong selection effect, formality rates increase dramatically. The aggregate formality rate increases from 66% to 91% without any change in tax rates or tax enforcement technologies. Output increases by more than 270%.

Table 12: Shift in the Distribution of Skills by One Standard Deviation

	Percentile Interval	Benchmark	Shift by σ in Distribution of Entrepreneurs	Workers	Both
Average wage	0-25	1	1	1	1
(normalized by first quartile)	25-50	1.83	1.68	1.1	1.1
	50-75	3.07	2.66	1.23	1.24
	75-100	6.23	5.22	1.6	1.62
Share of Total Payroll	0-25	0.030	0.046	0.030	0.036
by Percentile Interval	25-50	0.056	0.065	0.064	0.072
	50-75	0.098	0.126	0.111	0.096
	75-90	0.133	0.146	0.170	0.185
	90-95	0.117	0.104	0.113	0.123
	95-100	0.566	0.512	0.512	0.488
Formality Rate	0-25	0	0	0.360	0.354
by Percentile Interval	25-50	0	0	0.528	0.485
	50-75	0.062	0.038	0.728	0.638
	75-90	0.324	0.254	0.952	0.883
	90-95	0.586	0.497	1	1
	95-100	0.951	0.908	1	1
	Aggregate	0.656	0.559	0.912	0.883
Fraction of Entrepreneurs		0.231	0.301	0.118	0.148
Workers share of income		0.628	0.627	0.713	0.723
% change in wages wrt benchmark		0	0.118	-0.038	0.047
% change in output wrt benchmark		0	0.075	2.731	2.957

Note: The last three columns report the effect of changing the distribution of skills by adding a standard deviation to the abilities of agents only as entrepreneurs, only as workers , and both. Bins for wages are created by ordering employees by wages. Bins for firms are created by ordering them by the size of their payroll. The bins for the size distribution of firms and formality rates are the same.

The third exercise, in which all abilities are raised by a standard deviation, is qualitatively similar to the second one, but produces more small entrepreneurs than the former case. This can be seen in figure 7 where we observe more entrepreneurs clustered around the average income. The fraction of entrepreneurs in the population increases to 15%. Comparing the third and second exercise is similar to the comparison between the first one and the benchmark in which only entrepreneurial talent increased.

We now turn to the second batch of exercises in which we increase the variance of entrepreneurial and worker talents, one at a time and then both simultaneously. The motivation is the observation from figure 2 that cognitive abilities are much more concentrated in Mexico. The ratio of the standard deviation of PISA scores between the US and Mexico ranges from 1.14 in reading to 1.27 in science.

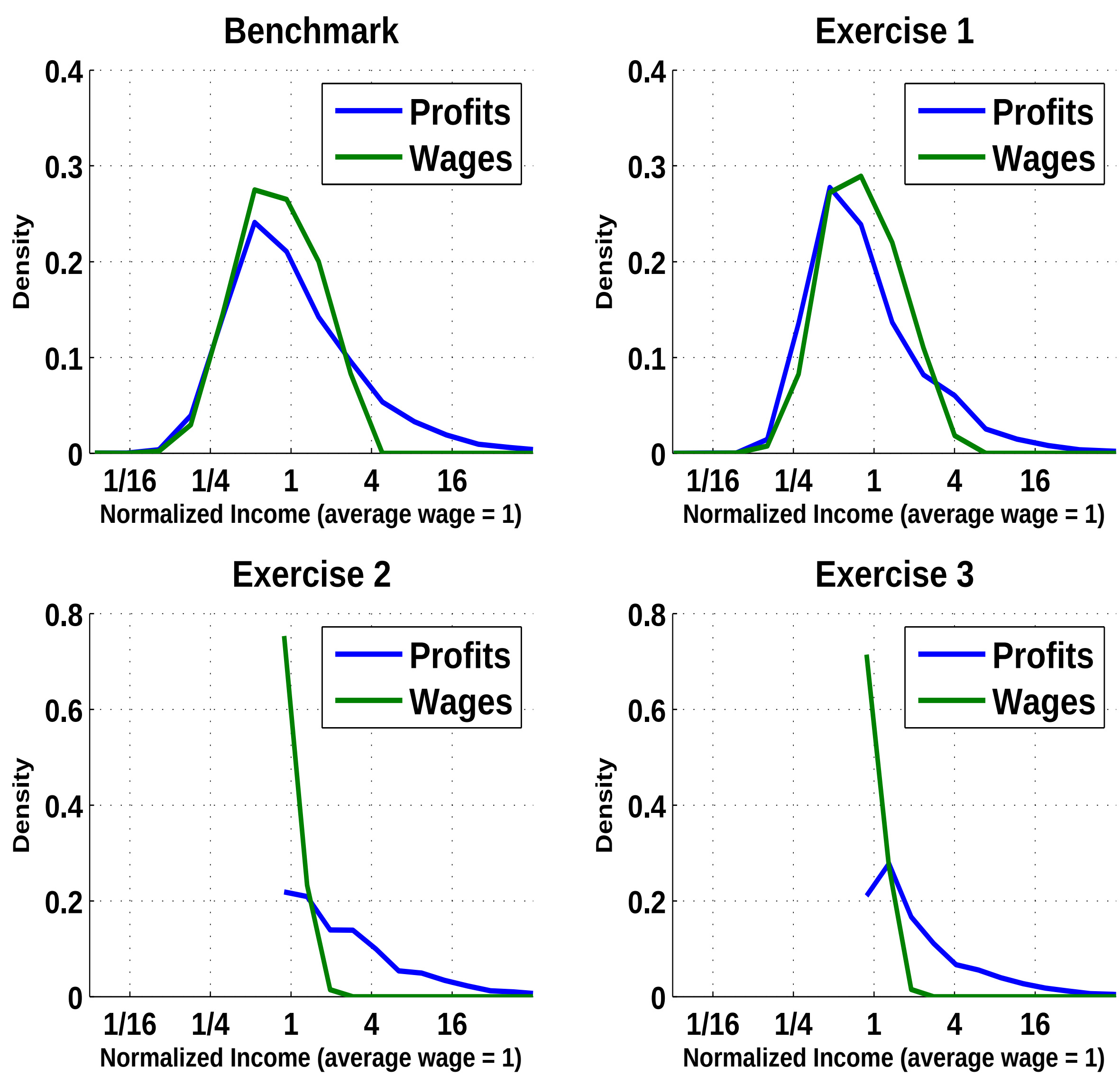
In the first of this set of experiments we increase the variance of entrepreneurial talents. This has the effect of making the tails of the distribution much worse on the left and much better on the right. Table 13 shows that the fraction of entrepreneurs in the population drops from 23% to 14%. The remaining entrepreneurs demand more labor and wages go up by 180% in spite of the increase in the labor supply. The selection of entrepreneurs leads to larger firms and an increase in formality rates. The aggregate formality rate increases from 66% to 81%. The relative wage of the top quartile of workers relative to the bottom one increases as many entrepreneurs on the left tail now opt to become low income workers. As a result of all these effect output increases by almost 190%.

The second exercise increases the standard deviation of working ability keeping entrepreneurial talent constant. This spread in working abilities means that agents with low skills as workers, now become even worse and decide to become entrepreneurs. At the top of the distribution, working talent improves and less people decide to be entrepreneurs. The remaining entrepreneurs at the top of the distribution have higher z_e's by Proposition 3. These results in an increase in the fraction of entrepreneurs in the population to 27% and no effect on wages. Labor is drawn to larger firms and formality rates slightly increase. The increase in the dispersion of working talent also leads to more dispersed wages. Output increases by 25%.

The third exercise increases the variance of both talent distributions by 30%. As it was the case with the shifts in the distribution, qualitatively the change in the equilibrium from the benchmark case to the case in which only the variance of entrepreneurial is more dispersed is similar to the case in which we add entrepreneurial talent dispersion to the case in which we increases worker talent dispersion above.

The last case we consider is a shift in the distribution of entrepreneurial and working talent by their respective standard deviations as well as an increase in the variance of each distribution by 30%. The dominant force here is the shift in working talent that generates the dramatic change in income distribution that can be seen in the panel for Exercise 3 in figure 7 and in the panel for Exercise 7 in figure 8. The combination of this effect with the selection effect on entrepreneurs described in the exercise in which the standard deviation of the talent of entrepreneurs was increased by 30% produces a huge effect on output, which increases more than 10 times. The fraction of entrepreneurs in the population drops to 7.5%, a magnitude that is very close to the 6.9% reported by ILO for the United States in 2008.

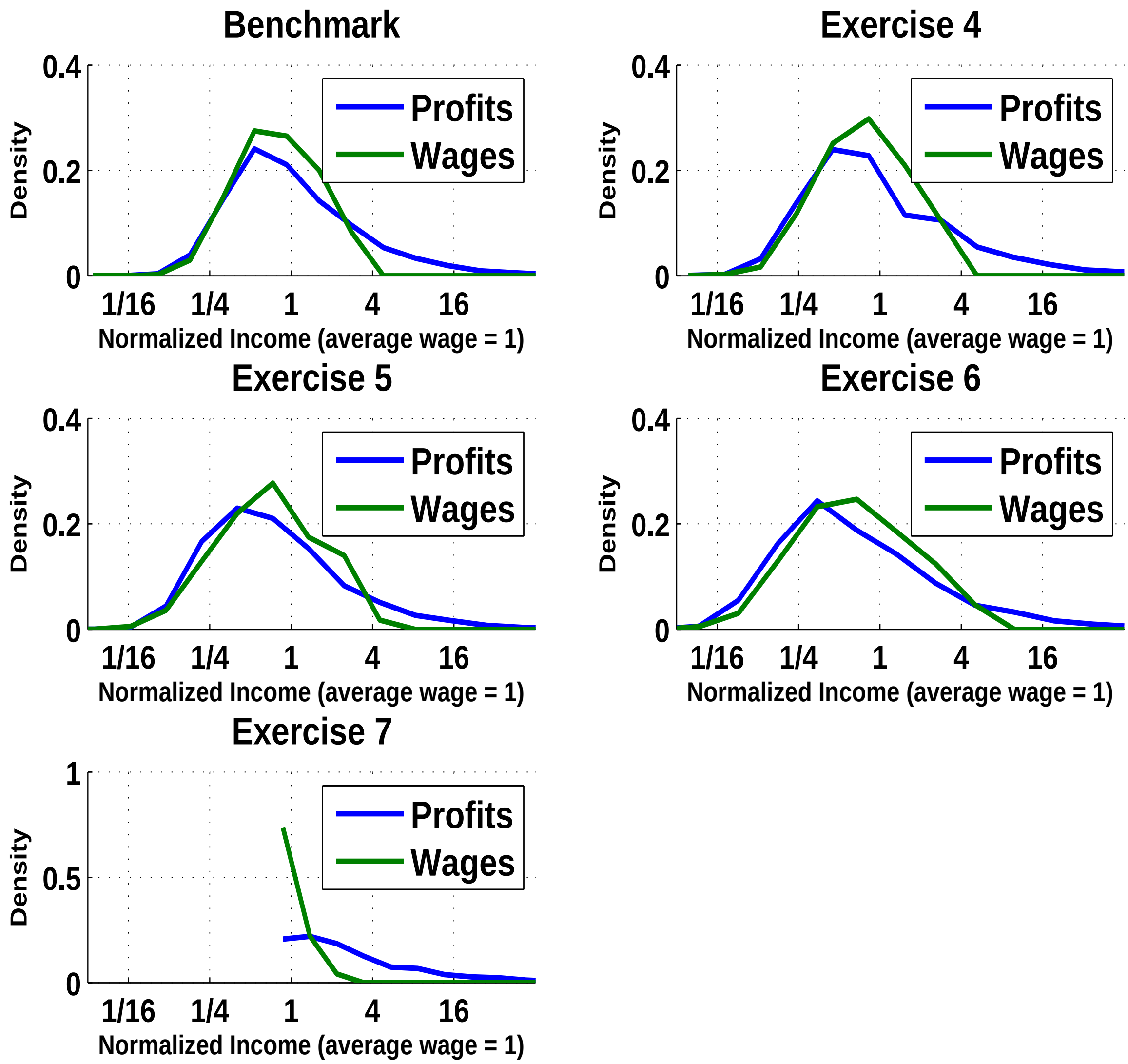
Figure 7: Shift in the Distribution of Skills by One Standard Deviation



Note: Exercise 1 shifts the distribution of z_e by σ_e. Exercise 2 shifts the distribution of z_w by σ_w. Exercise 3 shifts the distribution of z_e and z_w by σ_e and σ_w, respectively.

Finally, we look at how formality rates change when we shift the skill distribution for all the population. In Section 1 we looked at how informality rates vary with cognitive skills in Latin America. The results where reported in figure 3. Unfortunately we cannot ask our model to reproduce figure 3 since the number of workers of each firm in the model is indeterminate. The model, however, allows us to look at how the informal fraction of the wage bill (not the number of workers) varies with the population's talent bundles. Figure 9

Figure 8: Spread in the Distribution of Skills



Note: Exercise 4 increases σ_e by 30%. Exercise 5 increases σ_w by 30%. Exercise 6 increases both σ_e and σ_w by 30%. Exercise 8 increases all values of z^e and z^e by σ_e and σ_w, respectively and the two standard deviations by 30%

Table 13: Spread in the Distribution of Skills

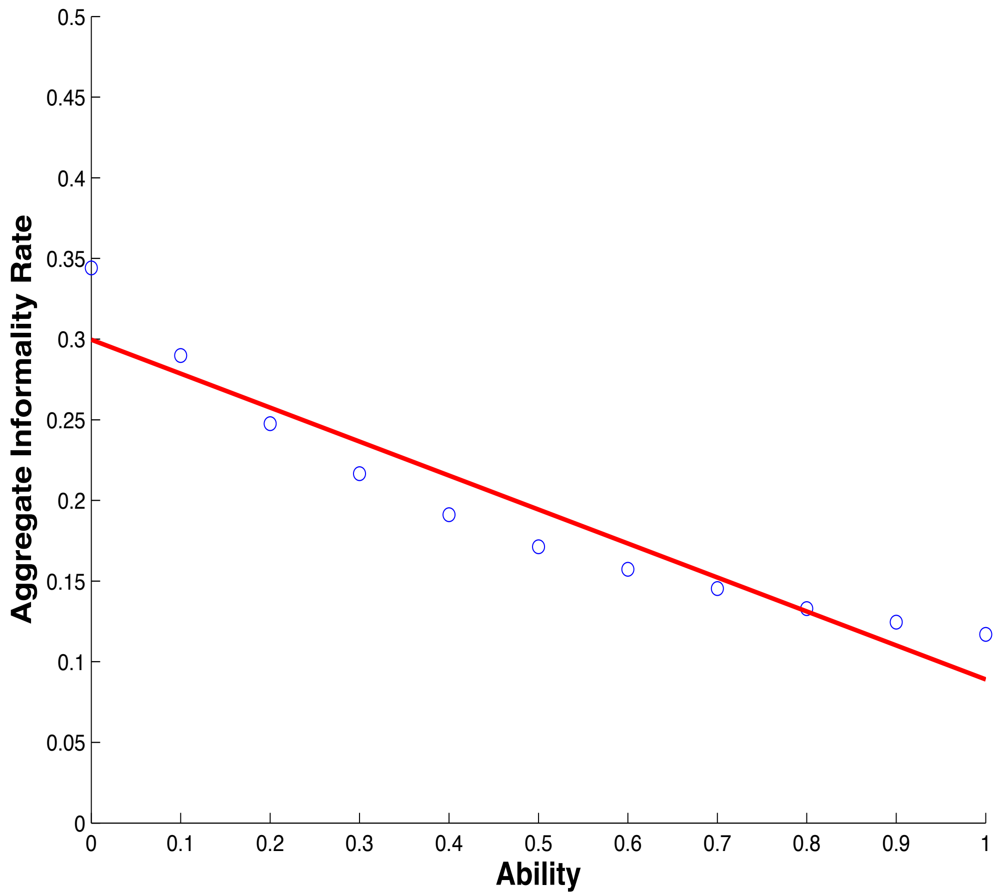
	Percentile Interval	Benchmark	Increase in σ by 30% for Entrepreneurs	Workers	Both	Shift in Mean and σ all
Average wage	0-25	1	1	1	1	1
(normalized by	25-50	1.83	1.91	2.11	2.24	1.1
first quartile)	50-75	3.07	3.31	3.97	4.38	1.29
	75-100	6.23	7.09	9.65	11.48	1.95
Share of Total	0-25	0.030	0.013	0.025	0.011	0.016
Payroll by	25-50	0.056	0.028	0.046	0.023	0.026
Percentile Interval	50-75	0.098	0.044	0.1	0.042	0.077
	75-90	0.133	0.1	0.139	0.094	0.125
	90-95	0.117	0.085	0.103	0.079	0.102
	95-100	0.566	0.73	0.587	0.75	0.654
Formality Rate	0-25	0	0	0	0	0.354
by Percentile	25-50	0	0	0	0	0.499
Interval	50-75	0.062	0.017	0.071	0.015	0.721
	75-90	0.324	0.328	0.361	0.318	0.974
	90-95	0.586	0.654	0.614	0.65	1
	95-100	0.951	0.985	0.957	0.986	1
	Aggregate	0.656	0.808	0.682	0.821	0.952
Fraction of Entrepreneurs		0.231	0.143	0.268	0.172	0.075
Workers share of income		0.628	0.645	0.629	0.644	0.692
% change in wages wrt benchmark		0	1.792	0.008	1.799	1.783
% change in output wrt benchmark		0	1.893	0.247	2.575	10.917

Note: Columns 4-6 report the effect of increasing the standard deviation of skills by 30% only for entrepreneurs, only for workers , and for both. Column 7 reports the effect of shifting the distribution by one standard deviation and increasing the standard deviation by 30%. Bins for wages are created by ordering employees by wages. Bins for firms are created by ordering them by the size of their payroll. The bins for the size distribution of firms and formality rates are the same.

plots this measure of informality against talent bundles. Each dot represents the informality rate in the model after shifting abilities by the corresponding fraction of a standard deviation. We see that there are striking similarities between figure 3 and figure 9 [13]. Figure 3 looks noisier than figure 9 because, among other things, in the latter the dispersion of skills as well as labor market policies are kept constant.

[13]Generating the data analog of figure 9 would require to have the census micro data for each country to calculate the fraction of unpaid social security contributions

Figure 9: Shift in the Skill Distribution and Informality



Note: The aggregate informality rate in the vertical axis is the *informal* fraction of the wage bill. In the horizontal axis, 0 corresponds to the benchmark calibration and 1 to increasing the working and entrepreneurial talents by a standard deviation, respectively. Each circle in the diagram reports the informality rate in the model after shifting abilities by the corresponding fraction of a standard deviation.

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