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Abstract

We study the effect of UI benefits in a typical developing country where the informal sector is sizeable and persistent. In a partial equilibrium environment, ruling out the macroeconomic consequences of UI benefits, we characterize the stationary equilibrium of an economy where policyholders may be employed in the formal sector, short-run unemployed receiving UI benefits or long-run unemployed without UI benefits. We perform comparative static exercises to understand how UI benefits affect unemployed workers' effort to secure a formal job, their labor supply in the informal sector and leisure time. Our model reveals that an increase in UI benefits generates two opposing effects for the short-run unemployed. First, since search efforts cannot be monitored it generates moral hazard behaviours that lower effort. Second, it generates an income effect as it reduces the marginal cost of searching for a formal job and increases effort. The overall effect is ambiguous and depends on the relative strength of these two effects. Additionally, we show that an increase in UI benefits increases the efforts of long-run unemployed workers. We provide a simple simulation exercise which suggests that the income effect pointed out is not necessarily of second-order importance in comparison with moral hazard strength. This result softens the widespread opinion, usually based on the microeconomic/partial equilibrium argument that the presence of dual labor markets is an obstacle to providing UI in developing countries.

Key words: Unemployment insurance, informal sector, income effect, developing countries.

JEL Classification Numbers: H55, I38 and J65.

1 Introduction

In spite of a large body of literature dealing with unemployment insurance (UI) in recent decades, few studies have analyzed the consequences of UI benefits on labor markets with a substantial informal sector. Indeed, developing countries' dual labor markets may reduce the desirability of a UI program. As pointed out in Hopenhayn and Nicolini (1999) and Alvarez-Parra and Sanchez (2009),[1] the incentives problem becomes much stronger if the state is not able to control the status of the unemployed, that is, if the unemployed work in the informal sector while receiving UI benefits. This pessimistic view is eloquently expressed in Mazza (2000): "The preliminary evidence gathered from Latin American and Eastern European cases is that the presence of a large informal sector may undermine the utility of UI, by making it impossible to insure that recipients are looking for new work, and may provide perverse incentives to increase further the informal sector...much more systematic study is needed and recommended by this study before firmer conclusions can be drawn." However, receiving UI benefits also generates an income effect and may allow the unemployed to devote less time in remunerated informal activities and consequently devote more time to secure a job in the formal sector. It is difficult to predict the net effect of increasing UI benefits on the time devoted to secure a formal-sector job because of the presence of both the traditional moral hazard effect and the income effect.

In addition, there are other macroeconomic forces at play that may affect the impact of UI benefits on the time devoted to secure a formal sector job in countries with a substantial informal sector. In particular, an increase of UI benefits may increase the bargaining power of formal-sector workers which in turn increases their wage. This wage increase may, on the one hand, increase the gap between wage and workers' productivity and therefore contribute to increase unemployement. On the other hand, in a dual labor market, this wage increase makes the formal sector more attractive as compared to the informal sector, *ceteris paribus.*

Governments and multilateral agencies are reluctant to adopt or promote sizeable UI schemes in dual labor markets partly because of the the moral hazard issue described in Mazza (2010). In this paper we focus on studying the effect of increasing UI benefits in the presence of both moral hazard and income effect on the time devoted to secure a

[1] To our knowledge, only Alvarez-Parra and Sanchez's (2009) analysis formally deals with the consequences of UI in the presence of a 'hidden market'. Their article adopts a sophisticated mechanism design approach in order to characterize the optimal dynamic UI contract in a partial equilibrium set up. The authors design the optimal UI contract to avoid the emergence of an informal sector for as long as possible, which in their set up would be generated by flaws in UI benefit design. This type of analysis is clearly relevant for developed countries where hidden labor markets are relatively small and can be largely explained by UI coverage. They point out that the optimal unemployment insurance system -in an economy with a potentially sizeable hidden market- is characterized by a relatively flat payment schedule for the short-run unemployed, and no payments for the long-run unemployed.

formal-sector in a partial equilibrium set up. Our aim is to disentangle analytically the different effects at work in this partial equilibrium environment in order to address the UI consequences on dual labor markets.

We build up a partial equilibrium environment, ruling out the macroeconomic consequences of UI benefits, where the informal sector is sizeable, persistent and the bulk of it cannot be explained by UI benefits.[2] We then take as given the presence of a dual labor market and characterize the stationary equilibrium of an economy where policyholders may exist in three different states: Employed in the formal sector, short-run unemployed receiving UI benefits or long-run unemployed without UI benefits but receiving an unemployment subsidy from the state. While the labor supply of formal workers is assumed to be inelastic, unemployed workers (short or long-run) can divide their total time between searching for a formal job (effort), working in the informal sector and enjoying leisure time. Moreover, following Fredriksson and Holmlund (2001) we adopt a duration framework in which short-run unemployed workers may lose UI benefits at a stochastic rate.[3] Instead of determining the optimal UI contract we perform comparative static exercises of the stationary equilibrium in order to understand how the unemployment policy (UI benefits, unemployment subsidy and rate of expiration of UI eligibility) affect unemployed workers' short and long-run decisions, that is, effort in securing a new job, informal labor supply and leisure time. We assume that none of these variables can be observed by the UI provider in order to tackle moral hazard issues.

Our model suggests that in an economy with a substantial informal sector an increase in UI benefits (subsidies) generates two countervailing effects on the short-run (resp. long-run) unemployeds' efforts when searching for a formal job. On the one hand, as the UI provider cannot perfectly monitor agents' search efforts,[4] moral hazard behaviours tend to lower them. Interestingly, moral hazard behaviours have two components. First, the time devoted in order to secure a new job in the formal sector or to leisure activities has an opportunity cost in the form of informal sector work, which reduces the unemployed workers' search efforts. Second, there is also a more standard moral hazard effect which increases the unemployed workers' leisure time and may decrease the search effort. We show that when consumption and leisure are complementary leisure increases, but that it does so at the expense of labor supply in the informal sector rather than at the expense of effort. On the other hand, UI benefits generate an income effect as it reduces the marginal cost of searching for a formal job. This income effect in-

[2] Developing countries are characterized by the presence of informal labor markets; few of them offer formal coverage against the risk of unemployment.

[3] For simplicity, we assume away the possibility of smoothing consumption through borrowing and saving.

[4] Note that in our setting the UI provider can indifferently be part of the public sector or a private insurer.

creases unemployed workers' efforts at the expense of their labor supply in the informal sector and therefore softens the moral hazard issue that arises from the unobservability of effort.[5] The overall effect of an increase in UI benefits on the efforts of short-run unemployed workers is generally ambiguous and depends on the relative strength of these two effects. Nevertheless, using a constant relative risk aversion utility function as in Fredriksson and Holmlund (2001) and Hansen and Imrohoroğlu (1992), we derive a sufficient condition which ensures that effort undertaken by short run unemployed workers increases with UI benefits.

We also show that an increase in UI benefits received by short-run unemployed workers unambiguously increases the efforts of long-run unemployed workers to find a formal job. This is because an increase in UI benefits increases the marginal benefit of effort. This result has strong consequences for economic policy: In addition to providing coverage against the risk of unemployment, UI may be a tool for reducing the size of the informal sector in developing countries (entitlement effect), so long as the informal sector is mainly composed of long-run unemployed workers. Concerning the expected duration of UI benefits, our results reveal an intertemporal trade-off. Roughly speaking, when UI duration increases, *ceteris paribus*, it raises the moral hazard issue, decreasing the effort of the short-run unemployed. However, the long-run unemployed workers' effort increases with UI (expected) duration. Again, this result suggests that the optimal duration of UI benefits in a developing country may depend on the fraction of long-run unemployed and the average length of unemployment episodes. Finally, we provide simple numerical simulation exercises in order to work out the potential size of the moral hazard and income effects that we characterize analytically. Our results suggest that the income effect is not necessarily of second-order importance in comparison with the moral hazard strength. This implies that at least in a partial equilibrium environment, an increase in UI benefits may increase unemployed workers' efforts to secure a job in the formal sector, instead of increasing their labor supply in the informal sector. This result softens the widespread view that the presence of dual labor markets is an obstacle to providing UI in developing countries. It also suggests that a general equilibrium analysis including this moral hazard/income effect tension could shed light on the overall effects of increasing UI benefits in countries with a substantial informal sector.

The next section presents the model and its analytical results. Section 3 is devoted to some numerical simulations. We conclude and provide some policy recommendations in section 4.

[5] This effect is close to the liquidity constraint pointed out by Chetty (2008).

2 The Model

We construct a continuous time model in order to analyze the effects of increasing UI in an economy characterized by a significantly sized informal sector. Workers can be either employed in the formal sector or unemployed. When they are employed in the formal sector they receive an hourly wage equal to w^f. Formal-sector jobs are destroyed at a rate ϕ, and workers become unemployed.[6] Unemployed agents can either be short or long-run unemployed. When workers lose a formal-sector job they become short-run unemployed (denoted by index $j = I$) and receive UI benefits. Following Fredriksson and Holmlund (2001), we assume that UI benefits may expire at a Poisson rate,[7] λ, independent of the policyholders' actions. This implies that the expected duration of UI benefits equals $1/\lambda$. When UI benefits expire agents become long-run unemployed, $j = N$, and they do not receive UI benefits anymore, instead they receive a transfer referred to as subsidy. Formal-sector opportunities arrive at rate p^I for the short-run unemployed and p^N for the long-run unemployed.

When employed in the formal sector we assume that workers split their total time, T, between formal-sector work, H, and leisure, $L = T - H$. Since we want to focus on the consequences of increasing UI benefits on the decisions of unemployed workers, we suppose that the number of hours worked in the formal sector are exogenous. In contrast, when unemployed, either short or long-run, agents split their total time, T, into three activities. First, they can devote s^j units of time to secure a formal-sector job, called effort hereafter. Second, they can work a^j units of time in the informal sector to earn an income. Finally, they can enjoy l^j units of leisure time. The time constraint is

$$s^j + a^j + l^j = T.$$

The total time that an unemployed worker devotes to the informal sector is then given by $T - s^j - l^j$. Crucially, we assume that s^j and a^j cannot be observed, that is, they are part of the private information of the unemployed workers and consequently are not contractible. Moreover, effort affects the rate $p^j(s^j)$ at which workers find a formal job, with $p'(.) > 0$ and $p''(.) < 0$. Finally, when working in the informal sector, which is assumed to be frictionless and without rationing, workers receive an hourly wage of $w^i = kw^f$, where $0 \leq k < 1$. We assume that there exists a positive differential of wages between the formal and informal sectors.[8]

[6] The financial market is supposed to be imperfect, that is, there are not any financial assets that allow workers to be covered against the risk of losing their job.

[7] The usual caveat applies for using a duration set up. As pointed out in Mortensen (1977), we avoid dealing with "the issue of how search effort changes over the spell of insured unemployment."

[8] This wage differential can be explained in several ways. For instance, in formal firms there are on-the-job training programs that enhance the human capital of formal workers. Even though it is out of the scope of this model, there may also be a self-selection effect,that is, more qualified workers tend

2.1 Workers

Agents are risk-averse and their preferences are represented by an increasing and concave VNM utility function, u. Let V^e be the value of formal-sector employment, V^I the value of the short-run unemployed workers who enjoy UI benefits and V^N the value of the long-run unemployed workers who no longer have access to UI but benefit from a UI subsidy. The flow value of a formal-sector job is

$$rV^e = u(w^f H, T - H) - \phi \left[V^e - V^I\right], \tag{1}$$

where r denotes the subjective rate of time preference. The flow value of a formal job depends on the income obtained and the leisure time enjoyed. A formal worker loses his job with probability ϕ and in this case becomes a short-run unemployed facing a capital loss of $V^e - V^I$.

The short-run unemployed receive UI benefits of $bw^f H$, where b denotes the replacement ratio. While receiving UI benefits she can work in the informal sector a^I units of time, where she earns an income of $kw^f a^I$. She can also exert effort (s^I) to secure a formal job with probability $p^I(s^I)$, thus realizing a capital gain of $V^E - V^I$. With probability λ, the short-run unemployed becomes a long-run unemployed, loses the UI benefits, and thus faces a capital loss of $V^I - V^N$. Therefore, the value function of a short-run unemployed is

$$rV^I = u^I(kw^f a^I + bw^f H, T - s^I - a^I) + p^I(s^I)\left(V^E - V^I\right) - \lambda\left(V^I - V^N\right). \tag{2}$$

Similarly, the flow value of being long-run unemployed, without access to UI benefits, is

$$rV^N = u^N(kw^f a^N + zw^f H, T - s^N - a^N) + p^N(s^N)\left(V^E - V^N\right). \tag{3}$$

Long-run unemployed workers earn $kw^f a^N$ from their labor supply in the informal sector and also benefit from a government transfer,[9] $zw^f H$. We naturally assume that $b > z$.

Considering the government's instrument (b, z) as given, the unemployed workers choose $\left(s^j, l^j, a^j\right)$, such that

$$\left(s^j, l^j, a^j\right) \in \arg\max V^j.$$

The first order conditions of this maximization program yield

$$-\frac{\partial u(kw^f a^j + bw^f H, T - s^j - a^j)}{\partial l^j} + \frac{\partial p^j(s^j)}{\partial s^j}\left[V^E - V^j\right] = 0 \tag{4}$$

to prefer the formal sector.

[9] z is a fraction of their last wage and can be thought of as "social assistance".

and

$$kw^f \frac{\partial u(kw^f a^j + bw^f H, T - s^j - a^j)}{\partial c^j} - \frac{\partial u(kw^f a^j + bw^f H, T - s^j - a^j)}{\partial l^j} = 0. \quad (5)$$

Equation (4) shows that an unemployed worker undertakes effort to secure a new job in the formal sector such that the marginal benefit of this effort, composed by the marginal increase of the probability of finding a job, times the difference of values between being employed ($j = E$) and unemployed ($j = I, N$), is equal to the marginal cost due to the reduction of leisure. Equation (5) shows that an unemployed worker chooses his level of informal labor supply to equalize his marginal consumption utility to his leisure marginal (opportunity) cost.

In the following section we provide comparative static analysis that allows us to disentangle the different effects of UI benefits and unemployment subsidies on unemployed workers' decision variables in this partial equilibrium environment.

2.2 Comparative Statics at the Stationary Equilibrium

Similarly to Fredriksson and Holmlund (2001), we combine (1), (2) and (3) and obtain at the stationary equilibrium:

$$\begin{aligned}
V^E - V^I &= \frac{1}{A}\left[\left(r + p^N(s^N)\right)\left[u(w^f T) - u^I\left(c^I, l^I\right)\right] + \lambda\left[u(w^f T) - u^N\left(c^N, l^N\right)\right]\right], \\
V^E - V^N &= \frac{1}{A}\left[\left(r + \lambda + p^I(s^I)\right)\left[u(w^f T) - u^N\left(c^N, l^N\right)\right] + \phi\left[u^I\left(c^I, l^I\right) - u^N\left(c^N, l^N\right)\right]\right], \\
V^I - V^N &= \frac{1}{A}\left[\left(r + \phi + p^I(s^I)\right)\left[u^I\left(c^I, l^I\right) - u^N\left(c^N, l^N\right)\right] + \left(p^I(s^I) - p^N(s^N)\right)\left[u(w^f T) - u^I\left(c^I, l^I\right)\right]\right],
\end{aligned}$$

where $A = \left(r + p^N(s^N)\right)\left(r + \phi + p^I(s^I)\right) + \lambda\left(r + \phi + p^N(s^N)\right)$. In what follows, we substitute the term $V^E - V^j$ we get at the stationary equilibrium into the first order conditions of the short and long-run unemployed workers. We then perform several comparative statics exercises.

First, let us analyze the effects generated by increasing UI benefits (respectively UI subsidies) on decisions taken by short-run (resp. long-run) unemployed workers.

Proposition 1 *For short-run (long-run) unemployed workers an increase in b (resp. in z) has ambiguous effects on informal-sector work, a^I (resp. a^N), and time devoted to searching for a formal-sector job, s^I (resp. s^N); if $u_{cl} \geq 0$ it increases leisure, l^I (resp. l^N).*

Proof. Appendix. ∎

Let us interpret the results for short-run unemployed workers, the intuition being the same for the results of the long-run unemployed. At the stationary equilibrium the first order conditions of the unemployed workers contain a wealth effect, which mainly occurs at an intratemporal level, and a moral hazard effect, which captures the effects of the next period policy variables on the unemployed workers' decisions.

Let us first consider the condition that determines the effect of UI benefits on short-run unemployed workers' effort s^I,

$$\frac{ds^I}{db} = \frac{w^f H}{|G^I|}\left[kw^f\left(u^I_{cc}u^I_{ll} - \left(u^I_{lc}\right)^2\right) + \frac{\left(r + p^N(s^N)\right)\partial p^I(s^I)/\partial s^I}{A}u^I_c\left[G^I_{aa}\right]\right],$$

where $\left|G^I\right|$ and G^I_{aa} are positive and negative respectively due to the second order conditions (see appendix for more details). The first term ($kw^f\left(u^I_{ll}u^I_{cc} - \left(u^I_{cl}\right)^2\right) > 0$) captures the wealth effect generated by the UI benefits: Thanks to UI benefits, all else being equal, short-run unemployed workers need to spend less time working in the informal sector and can devote more time to securing a formal-sector job. The second term is due to the presence of moral hazard: An increase in UI benefits in the future reduces $\left(V^E - V^I\right)$, thus weakening incentives to secure a job in the formal sector. The existence of these two countervailing effects generates the ambiguous results for the search effort summarized in Proposition 1.

Remark 1 *When $u = c^\sigma L^{\sigma\delta}/\sigma$ and assuming that $p^I(s^I) = s^I\alpha$, then the following sufficient condition ensures that $ds^I/db \geq 0$:*

$$\left[1 - (1+\delta)\sigma\right]\left[r + \phi + \lambda\right] > \alpha\left(T + \frac{bw^f}{w^i}\right).$$

Proof. See appendix. ■

When the condition in Remark (**??**) is satisfied UI benefits increase the short run unemployed workers' effort. This condition is more likely to be satisfied when the risk aversion (σ) and formal employment destruction rate (ϕ) are high,[10] the average duration of UI benefits ($1/\lambda$) and the relative earnings in the formal and informal sectors ($\frac{w^f}{w^i}$) are low. On the other hand, it is less likely that this sufficient condition is satisfied when the probability of finding a formal employment is very sensitive to (short run) unemployed workers' effort (α).

Let us now turn to the effect of UI benefits on the time devoted by short-run unemployed workers to informal activities. The effect of UI benefits on the informal-sector

[10] Note that $1 - (1+\delta)\sigma > 0$ is ensured by the concavity of the utility function.

labor supply is given by

$$\begin{aligned}\frac{da^I}{db} &= \frac{w^f H}{|G^I|}\left[-kw^f\left(u_{ll}^I u_{cc}^I - \left(u_{cl}^I\right)^2\right) - \frac{\left(r + p^N(s^N)\right)\partial p^I(s^I)/\partial s^I}{A} u_c^I \left[G_{aa}^I\right]\right. \\ &\qquad \left. + kw^f u_c^I \left(kw^f u_{cc}^I - u_{lc}^I\right)\left[\frac{\left(r + p^N(s^N)\right)\partial p^I(s^I)/\partial s^I}{A} - S^I\right]\right],\end{aligned}$$

where $S^I = \left(\partial^2 p^I(s^I)/\partial\left(s^I\right)^2\right) / \left[\left(\partial p^I(s^I)/\partial s^I\right) A\right]$.

Interestingly, all else being equal, the same income effect that increases the short-run unemployed workers' effort also decreases the time devoted to informal activities (because of the negative sign preceding it). Moreover, the effect generated by moral hazard on the time devoted to informal-sector work can be divided into two components. The first is given by the second term and captures a moral hazard effect which increases short-run unemployed informal-sector work at the expense of effort. The second moral hazard effect is captured by the third term in the equation. It captures the trade-off between informal-sector work and leisure time. If leisure and consumption are complementary goods, that is, $u_{cl} \geq 0$, the income effect and the second moral hazard effect decrease the labor supply in the informal sector. The first moral hazard component is a countervailing effect as it increases informal-sector work. Therefore, the sign of da^I/db depends on the relative sizes of these effects. In section 3, we present some simulations which provide insights into the sign of the overall impact of increases in UI on the informal labor supply of the short-run unemployed.

Let us now determine the overall effect of UI benefits on leisure:

$$\begin{aligned}\frac{dl^I}{db} &= -\left(\frac{ds^I}{db} + \frac{da^I}{db}\right) \\ &= -\frac{w^f H}{|G^I|}\left[kw^f u_c^I \left(kw^f u_{cc}^I - u_{cl}^I\right)\left[\frac{\left(r + p^N(s^N)\right)\partial p^I(s^I)/\partial s^I}{A} - S^I\right]\right].\end{aligned}$$

First, it is worth noting that the income effect does not intervene in dl^I/db; this effect only plays a role in the unemployed workers' trade-off between s^I and a^I. Second, signing dl^I/db boils down to signing the expression $B \equiv kw^f u_{cc}^I - u_{lc}^I$. When consumption and leisure are complementary ($u_{cl} \geq 0$), an increase in unemployment insurance b unambiguously increases leisure time for the short-run unemployed. As described in the following remark, when $u_{cl} \leq 0$, this result still holds under a Cobb-Douglas specification.

Remark 2 *When $u = c^\sigma L^{\sigma\delta}/\sigma$, the sign of $B \equiv kw^f u_{cc}^I - u_{lc}^I$ is always negative and therefore $dl^I/db > 0$.*

Proof. Appendix. ■

Assuming a Cobb-Douglas utility function, we can establish the effect of an increase in unemployment insurance on leisure time, regardless of whether consumption and leisure are complements or substitutes: An increase in unemployment insurance always increases leisure time for the short-run unemployed. In this case, the second moral hazard component increases unemployed workers' leisure time at the expense of their labor supply in the informal sector.

Let us now turn to the effect of UI benefits on long-run unemployed workers. Note that we are back to the general case and do not assume a particular functional form of the utility function.

Proposition 2 *An increase in b unambiguously increases s^N; if $u^N_{cl} \geq 0$ it decreases a^N and increases l^N.*

Proof. See Appendix. ■

Interestingly, Proposition 2 reveals that UI benefits only generate a moral hazard effect on the short-run unemployed. UI benefits may decrease the unemployed workers' effort to secure a formal job while short-run unemployed. However, the existence of UI benefits received by the short-run unemployed unambiguously increases the effort undertaken by long-run unemployed workers to secure a formal job, s^N. This is because at the stationary equilibrium $V^E - V^N$ increases with $u^I - u^N$, which in turn increases with b. Finally, the effects of UI benefits on informal-sector work and leisure time of the long-run unemployed depend on the cross derivative between consumption and leisure. If consumption and leisure are complementary then again, an increase in UI benefits increases leisure at the expense of informal-sector work.

As revealed by Proposition 1, z affects the time allocation of the long-run unemployed in a similar way that b affects the decisions of short-run unemployed workers. In contrast, the following Proposition points out the effect of z on short-run unemployed workers' decisions.

Proposition 3 *An increase in z always decreases s^I; if $u_{cl} \geq 0$, it increases a^I and decreases l^I.*

Proof. See appendix. ■

Proposition 3 points out that social assistance generates a dynamic moral hazard effect on short-run unemployed workers' effort (s^I). An increase in z raises the utility u^N of long-run unemployed workers and, *ceteris paribus*, reduces short-run unemployed workers' efforts to secure a formal job. As usual, the effect of z on short-run unemployed workers' informal labor supply and leisure activity respectively increases and decreases when $u_{cl} \geq 0$ is assumed.

Finally, the expiration rate of UI benefits, λ, has a very close relationship with a key feature of UI design. The effects of changes in λ on the time allocation decisions of short and long-run unemployed workers are summarized in the following proposition.

Proposition 4 *An increase of λ:*
i) increases s^I; if $u^I_{cl} > 0$ it decreases a^I and increases l^I.
ii) decreases s^N; if $u^I_{cl} > 0$ it increases a^N and l^N.

Proof. See appendix. ■

An increase in the expiration rate of UI benefits (or a decrease in the duration of UI benefits) reduces the moral hazard effect for short-run unemployed workers since, *ceteris paribus*, they have greater incentives to secure a job in the formal sector. In this case, the trade-off between labor supply in the informal sector and leisure time is standard. Conversely, an increase of λ (decrease in the duration of UI benefits) raises the moral hazard effect for long-run unemployed workers. This is because $V^e - V^N$ decreases with λ so, all else being equal, an increase in λ decreases the marginal benefit of effort. Finally, when $u^N_{cl} > 0$ an increase in λ increases the labor supply in the informal sector and leisure of long-run unemployed workers, at the expense of time devoted to securing a formal-sector job.

In the next section we simulate the effect of an increase in UI benefits in an economy with a sizeable informal sector.

3 Simulation Exercises

Using parameter values which correspond to Brazilian labor market,[11] we provide numerical simulation exercises to shed light on the effect of increasing UI benefits. Figure (1) shows that increasing UI benefits decreases the time devoted to informal-sector work for both the short and long-run unemployed. Increasing UI benefits also increases the time devoted to securing a formal-sector job for the short and long-run unemployed. This implies that, even though in the model the effect of an increase in UI benefits is ambiguous, because there were several countervailing effects taking place, the simulation of the model suggests that the overall effect on informal-sector work is negative. In other words, increasing UI benefits in an economy with a sizeable informal sector may decrease the size of the informal sector, despite the moral hazard effects.

[11] See online appendix for more details on the calibration exercice.

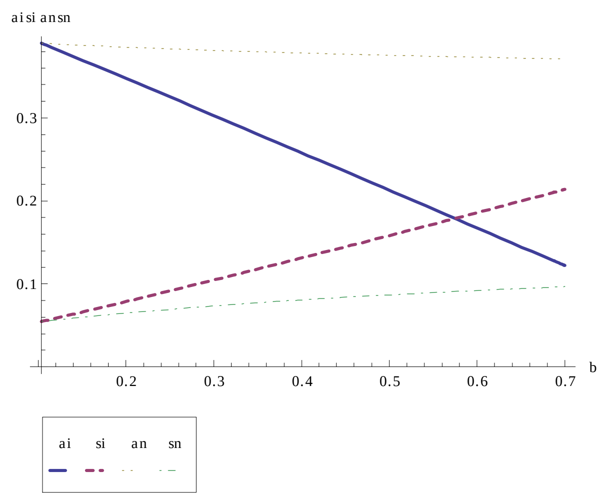

Figure 1. Increase in b

However, the effects of increasing UI benefits strongly depends on policyholders' risk aversion. Figure (2) repeats the exercise presented in Figure (2) but instead assumes that the risk-aversion coefficients are around the lower bound calculated by Cardenas and Carpenter (2010): $\gamma = 0.8$ ($\sigma = 0.27$). In this case, increasing b decreases the time devoted to informal-sector work for both the short and long-run unemployed, and it still increases the search efforts for the long-run unemployed. However, it decreases the search effort of the short-run unemployed.

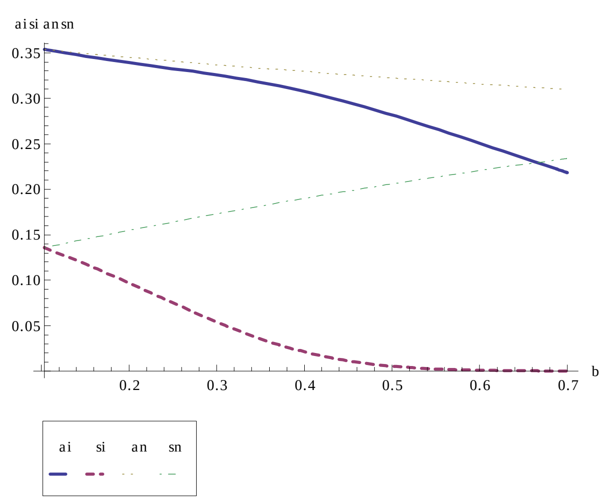

Figure 2. Increase in b

4 Discussion

The partial equilibrium set up allows us to derive analytical results on the consequences of increasing UI benefits on the moral hazard/income effect trade-off for unemployed workers in developing countries characterized by dual labor markets. Ruling out the general equilibrium effects, the first insight is that the standard moral hazard effect is common to all countries, that is, developed and developing countries, and it implies that the effort undertaken by unemployed workers decreases with UI benefits. Second, our analytical results point out that there is an income effect that offsets the moral hazard effect. Because UI benefits increase unemployed workers' incomes they need to devote less time to informal jobs and, *ceteris paribus,* they spend more time securing a new job in the formal sector. We then perform several simulation exercices. The results show that this income effect is not necessarily of second-order importance and may dominate the traditional moral hazard strength. Interestingly, the time devoted to searching for a formal-sector job of those no longer covered by UI benefits (the long-run unemployed) may increase with the level of UI benefits. Our results suggest that, contrary to the initial intuition,[12] increasing UI benefits in an economy with a sizeable informal sector may actually reduce the size of the informal sector by increasing the incentives of unemployed workers to secure a job in the formal sector.

Regarding the duration of UI benefits, our results highlight a trade-off between the incentives of short and long-run unemployed workers. Indeed, an increase in UI duration decreases short-run unemployed effort, whereas it reduces the moral hazard issue for long-run unemployed workers. In a partial equilibrium framework our results suggest that when choosing the duration of UI benefits policymakers should take into account the average duration of unemployment episodes in developing countries, as well as the composition of the unemployed (the proportion of short and long-run unemployed).

This paper aims to demonstrate that, first, there is an income effect generated by UI coverage which opposes the moral hazard effect and may even offset it. This suggests that developing countries should not be discouraged from adopting UI benefits by the mere existence of the moral hazard effect. Second, our comparative static results shed light on what could be the properties of an optimal UI contract in developing countries. This friendly partial equilibrium set up allows us to provide analytical results that are complemented by simulation exercises. However, to be able to characterize the optimal design of UI benefits in developing countries we strongly believe that this analysis must be extended in several ways. First, this issue could be resumed in a general equilibrium framework that would contain a matching process *a la* Pissarides in order to take into account the effect of UI coverage on the wage bargained in the formal sector. As it is likely that the design of optimal UI coverage depends on labor market features, this

[12] For instance, the one underlined in Mazza (2000).

general equilibrium approach should be combined with a calibration strategy using data from specific dual labor markets in developing countries. It is in our research agenda.

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6 Appendix: Proof of Propositions 1 2 3 and 4

Proof. This appendix is organized as follows: First, we calculate the Hessian matrices of both maximization programs, *i.e.* $j = \{I, N\}$. Next, we use them to provide the comparative static exercices that correspond to each proposition.

Let us define the following function $G^j = \left(G^j_{a^j}(a^j, s^j,), G^j_{s^j}(a^j, s^j,)\right) = (0, 0)$, $\forall j = \{I, N\}$.

First, let us focus on $j = I$. We have

$$\begin{aligned} G^I_{s^I} &= -\frac{\partial u(kw^f a^I + bw^f H, T - s^I - a^I)}{\partial l^I} \\ &\quad + \frac{\frac{\partial p^I(s^I)}{\partial s^I}}{A}\left[\left(r + p^N(s^N)\right)\left[u(w^f T) - u^I\left(kw^f a^I + bw^f H, T - s^I - a^I\right)\right]\right. \\ &\qquad \left. + \lambda\left[u(w^f T) - u^N\left(c^N, l^N\right)\right]\right], \end{aligned}$$

and

$$G^I_{a^I} = kw^f \frac{\partial u(kw^f a^I + bw^f H, T - s^I - a^I)}{\partial c^I} - \frac{\partial u(kw^f a^I + bw^f H, T - s^I - a^I)}{\partial l^I}.$$

We now calculate G^I_{ss}:

$$\begin{aligned} G^I_{ss} &= u^I_{ll} + \frac{1}{A^2}\left[A\left(\frac{\partial^2 p^I(s^I)}{\partial\left(s^I\right)^2}\left(V^e - V^I\right) + \frac{\partial p^I(s^I)}{\partial s^I}\left(r + p^N(s^N)\right)u^I_l\right)\right. \\ &\qquad \left. - \left(r + p^N(s^N)\right)\left(\frac{\partial p^I(s^I)}{\partial s^I}\right)^2\left(V^e - V^I\right)\right] \\ &= u^I_{ll} + \left[S^I\right]u^I_l, \end{aligned}$$

where $S^I = \left(\partial^2 p^I(s^I)/\partial\left(s^I\right)^2\right) / \left[\left(\partial p^I(s^I)/\partial s^I\right)A\right]$.

$$\begin{aligned} G^I &= \begin{pmatrix} G^I_{ss} & G^I_{sa} \\ G^I_{as} & G^I_{aa} \end{pmatrix} \\ &= \begin{pmatrix} u^I_{ll} + \left[S^I\right]u^I_l & -kw^f u^I_{lc} + u^I_{ll} \\ -kw^f u^I_{cl} + u^I_{ll} & \left(kw^f\right)^{2f} u^I_{cc} - 2kw^f u^I_{cl} + u^I_{ll} \end{pmatrix}. \end{aligned}$$

Notice that the second order conditions impose: $G_{ss} \leq 0$, $G_{aa} \leq 0$ and $\left|G^I\right| \geq 0$.

For $j = N$, we obtain

$$G_{a^N} = kw^f \frac{\partial u(kw^f a^N + zw^f H, T - s^N - a^N)}{\partial c^N} - \frac{\partial u(kw^f a^N + zw^f H, T - s^N - a^N)}{\partial l^N}$$

and

$$
\begin{aligned}
G_{s^N} &= -\frac{\partial u(kw^f a^N + zw^f H, T - s^N - a^N)}{\partial l^N} \\
&\quad + \frac{\frac{\partial p^N(s^N)}{\partial s^N}}{A}\left[\left(r+\lambda+p^I(s^I)\right)\left(u(w^f T) - u^N\left(kw^f a^N + zw^f H, T - s^N - a^N\right)\right)\right. \\
&\qquad \left. +\phi\left[u^I(kw^f a^I + bw^f H, T - s^I - a^I) - u^N(kw^f a^N + zw^f H, T - s^N - a^N)\right]\right].
\end{aligned}
$$

The Hessian matrix for the long-run unemployed, G^N, is given by:

$$
\begin{aligned}
G^N &= \begin{pmatrix} G^N_{ss} & G^N_{sa} \\ G^N_{as} & G^N_{aa} \end{pmatrix} \\
&= \begin{pmatrix} u^N_{ll} + S^N u^N_l & -kw^f u^N_{lc} + u^N_{ll} \\ -kw^f u^N_{cl} + u^N_{ll} & \left(kw^f\right)^2 u^N_{cc} - 2kw^f u^N_{cl} + u^N_{ll} \end{pmatrix}.
\end{aligned}
$$

■

6.1 Proof of Proposition 1

Proof. Applying the Cramer's rule yields:

$$
\begin{aligned}
\frac{da^I}{db} &= \frac{1}{|G^I|}\begin{bmatrix} G^I_{ss} & -G^I_{sb} \\ G^I_{as} & -G^I_{ab} \end{bmatrix} \\
&= \frac{1}{|G^I|}\begin{bmatrix} u^I_{ll} + \left[S^I\right] u^I_l & w^f H\left[u^I_{lc} + \frac{\left(r+p^N(s^N)\right)\partial p^I(s^I)/\partial s^I}{A} u^I_c\right] \\ -kw^f u^I_{cl} + u^I_{ll} & -w^f H\left[kw^f u^I_{cc} - u^I_{lc}\right] \end{bmatrix} \\
&= \frac{w^f H}{|G^I|}\left[-kw^f\left(u^I_{ll}u^I_{cc} - \left(u^I_{cl}\right)^2\right) + \frac{\left(r+p^N(s^N)\right)\partial p^I(s^I)/\partial s^I}{A} u^I_c\left(kw^f u^I_{cl} - u^I_{ll}\right)\right. \\
&\qquad \left. + \left[S^I\right] u^I_l\left(u^I_{lc} - kw^f u^I_{cc}\right)\right].
\end{aligned}
$$

Similarly, we have:

$$
\begin{aligned}
\frac{ds^I}{db} &= \frac{1}{|G^I|}\begin{bmatrix} -G^I_{sb} & G^I_{sa} \\ -G^I_{ab} & G^I_{aa} \end{bmatrix} \\
&= \frac{1}{|G^I|}\begin{bmatrix} w^f H\left[u^I_{lc} + \frac{\left(r+p^N(s^N)\right)\partial p^I(s^I)/\partial s^I}{A} u^I_c\right] & -kw^f u^I_{lc} + u^I_{ll} \\ -w^f H\left[kw^f u^I_{cc} - u^I_{lc}\right] & \left(kw^f\right)^{2f} u^I_{cc} - 2kwu^I_{cl} + u^I_{ll} \end{bmatrix}
\end{aligned}
$$

After some simplifications, we obtain:

$$\frac{ds^I}{db} = \frac{w^f H}{|G^I|}\left[kw^f\left(u^I_{cc}u^I_{ll} - \left(u^I_{lc}\right)^2\right) + \frac{\left(r + p^N(s^N)\right)\partial p^I(s^I)/\partial s^I u^I_c}{A}\left[G^I_{aa}\right]\right].$$

Finally, we have:

$$\begin{aligned}\frac{dl^I}{db} &= \frac{ds^I}{db} + \frac{da^I}{db} \\ &= \frac{w^f H}{|G^I|}\left[u^I_c kw^f\left(kw^f u^I_{cc} - u^I_{cl}\right)\left[\frac{\left(r + p^N(s^N)\right)\partial p^I(s^I)/\partial s^I}{A} - S^I\right]\right].\end{aligned}$$

A sufficient condition to have l^I decreasing in b is $u^I_{cl} \geq 0$.

Similar computations yield for a^N, s^N and l^N. ■

6.2 Proof of Remark 1

Proof. In this CRRA case, we have $ds^I/db \geq 0$ if and only if

$$kw^f\left(\frac{u^I_{cc}u^I_{ll} - \left(u^I_{lc}\right)^2}{u^I_c}\right) + \frac{\partial p^I(s^I)/\partial s^I}{(r + \phi + p^I(s^I)) + \lambda\left(1 + \frac{\phi}{r + s^N\alpha(\theta)}\right)}\left[G^I_{aa}\right] \geq 0$$

$$\Leftrightarrow$$

$$[1 - (1+\delta)\sigma]\,\delta \geq \frac{\alpha(\theta)}{r + \phi + s^I\alpha(\theta) + \lambda\left(1 + \frac{\phi}{r + s^N\alpha(\theta)}\right)}(\delta + 1)\,L \geq 0$$

$$\Leftrightarrow$$

$$[1 - (1+\delta)\sigma]\,[r + \phi + \lambda]$$

$$\geq \alpha(\theta)\left(T + \frac{bH}{k}\right) - \alpha(\theta)\,s^I - [1 - (1+\delta)\sigma]\,\alpha(\theta)\,s^I - [1 - (1+\delta)\sigma]\,\lambda\frac{\phi}{r + s^N\alpha(\theta)}.$$

Then, a sufficient condition to ensure $ds^I/db \geq 0$ is:

$$[1 - (1+\delta)\sigma]\,[r + \phi + \lambda] > \alpha(\theta)\,T\left(1 + \frac{bH}{kT}\right).$$

■

6.3 Proof of Lemma 1

When $u = c^{\sigma} L^{\sigma\delta}/\sigma$ the sign of $\frac{dl^I}{db}$ is always negative.

Proof. First, it is obvious that B is negative if $u_{lc}^I > 0$. Then, let us focus on the case $u_{lc}^I < 0$. In such a case we have:

$$B \leq 0 \Leftrightarrow kw^f u_{cc}^I \leq u_{lc}^I.$$

From the first order condition, we know that

$$kw^f = \frac{u_l^I}{u_c^I}.$$

Moreover, note that for $u = c^{\sigma} L^{\sigma\delta}/\sigma$, we have $u_{cl} = \sigma\delta c^{\sigma-1} L^{\sigma\delta-1}$ iff $\sigma < 0$. Therefore the previous inequality becomes:

$$\begin{array}{rcl} u_l^I u_{cc}^I & \leq & u_{lc}^I u_c^I \\ & \Leftrightarrow & \\ \delta c^{\sigma} L^{\sigma\delta-1} (\sigma - 1) c^{\sigma-2} L^{\sigma\delta} & \leq & \sigma\delta c^{\sigma-1} L^{\sigma\delta-1} c^{\sigma-1} L^{\sigma\delta} \\ & \Leftrightarrow & \\ \sigma - 1 & \leq & \sigma. \end{array}$$

Q.E.D ■

6.4 Proof of Proposition 2

Proof. Applying the Cramer's rule gives:

$$\begin{aligned} \frac{da^N}{db} &= \frac{1}{|G^N|} \begin{bmatrix} G_{ss}^N & -G_{sb}^N \\ G_{as}^N & -G_{ab}^N \end{bmatrix} \\ &= \frac{1}{|G^N|} \begin{bmatrix} u_{ll}^N + S^N u_l^N & -\left(\frac{\frac{\partial p^N(s^N)}{\partial s^N} \phi w^f H}{A} u_c^I \right) \\ -kw^f u_{cl}^N + u_{ll}^N & 0 \end{bmatrix} \\ &= \frac{1}{|G^N|} \left[\left(\frac{\frac{\partial p^N(s^N)}{\partial s^N} \phi w^f H}{A} u_c^I \right) \left(-kw^f u_{cl}^N + u_{ll}^N \right) \right] \end{aligned}$$

Therefore, a sufficient condition to have $da^N/db \leq 0$ is $u_{cl}^N \geq 0$.

Similarly, we have

$$\begin{aligned}
\frac{ds^N}{db} &= \frac{1}{|G^N|}\begin{bmatrix} -G^N_{sb} & G^N_{sa} \\ -G^N_{ab} & G^N_{aa} \end{bmatrix} \\
&= \frac{1}{|G^N|}\begin{bmatrix} -\left(\frac{\frac{\partial p^N(s^N)}{\partial s^N})\phi w^f H}{A} u^I_c\right) & -kw^f u^N_{lc} + u^N_{ll} \\ 0 & G^N_{aa} \end{bmatrix} \geq 0
\end{aligned}$$

as $G^N_{aa} \leq 0$.

The effect of b on l^N is given by:

$$\begin{aligned}
\frac{dl^N}{db} &= -\left(\frac{ds^N}{db} + \frac{da^N}{db}\right) \\
&= -G^N_{sb}\left(G^N_{as} - G^N_{aa}\right) \\
&= kw^f G^N_{sb}\left(kw^f u^N_{cc} - u^N_{cl}\right),
\end{aligned}$$

which is positive if $u^N_{cl} \geq 0$.

Moreover, we have

$$\begin{aligned}
\frac{ds^I}{dz} &= \frac{1}{|G^I|}\begin{bmatrix} -G^I_{sz} & G^I_{sa} \\ -G^I_{az} & G^I_{aa} \end{bmatrix} \\
&= \frac{1}{|G^I|}\begin{bmatrix} w^f H \frac{\frac{\partial p^I(s^I)}{\partial s^I}\lambda}{A} u^N_c & -kw^f u^I_{lc} + u^I_{ll} \\ 0 & \left(kw^f\right)^{2f} u^I_{cc} - 2kwu^I_{cl} + u^I_{ll} \end{bmatrix} < 0.
\end{aligned}$$

■

6.5 Proof of Proposition 3

Proof. Comparative static with respect to z is given by:

$$\begin{aligned}
\frac{da^I}{dz} &= \frac{1}{|G^I|}\begin{bmatrix} G^I_{ss} & -G^I_{sz} \\ G^I_{as} & -G^I_{az} \end{bmatrix}, \\
&= \frac{1}{|G^I|}\begin{bmatrix} u^I_{ll} + \left[S^I\right] u^I_l & w^f H \frac{\frac{\partial p^I(s^I)}{\partial s^I}\lambda}{A} u^N_c \\ -kw^f u^I_{cl} + u^I_{ll} & 0 \end{bmatrix}, \\
&= -\frac{w^f H}{|G^I|}\left[\frac{\frac{\partial p^I(s^I)}{\partial s^I}\lambda}{A} u^N_c \left(u^I_{ll} - kw^f u^I_{cl}\right)\right].
\end{aligned}$$

Therefore, we obtain that $da^I/dz > 0$ if $u^I_{cl} > 0$. ■

6.6 Proof of Proposition 4

Proof. Comparative statics with respect to λ are given by:

$$\frac{da^I}{d\lambda} = \frac{1}{|G^I|}\begin{bmatrix} G^I_{ss} & -G^I_{s\lambda} \\ G^I_{as} & -G^I_{a\lambda} \end{bmatrix}$$

and,

$$\frac{ds^I}{d\lambda} = \frac{1}{|G|}\begin{bmatrix} -G^I_{s\lambda} & G^I_{sa} \\ -G^I_{a\lambda} & G^I_{aa} \end{bmatrix}.$$

As $G^I_{a\lambda} = 0$, first let us compute $G^I_{s\lambda}$. We have

$$\begin{aligned} G^I_{s\lambda} \quad = \quad & \frac{\frac{\partial p^I(s^I)}{\partial s^I}\left(r + p^N(s^N)\right)}{A^2}\left[\left[u(w^f T) - u^N\left(c^N, l^N\right)\right]\left(\left(r + \phi + p^I(s^I)\right)\right)\right. \\ & \left. - \left(r + \phi + p^N(s^N)\right)\left[u(w^f T) - u^I\left(kw^f a^I + bw^f H, T - s^I - a^I\right)\right]\right] \end{aligned}$$

Therefore, we have

$$\begin{aligned} G^I_{s\lambda} & \geq 0 \\ & \Leftrightarrow \\ \frac{u(w^f T) - u^N\left(c^N, l^N\right)}{u(w^f T) - u^I\left(kw^f a^I + bw^f H, T - s^I - a^I\right)} & \geq \frac{\left(r + \phi + p^N(s^N)\right)}{\left(r + \phi + p^I(s^I)\right)}, \end{aligned}$$

which is the satisfied as long as $p^N(s^N) < p^I(s^I)$. Consequently, we have:

$$sign\left(\frac{da^I}{d\lambda}\right) = sign\left(-kw^f u^I_{cl} + u^I_{ll}\right).$$

We have $da^I/d\lambda < 0$ if $u^I_{cl} > 0$.

$$\frac{ds^I}{d\lambda} = \frac{1}{|G|}\begin{bmatrix} -G^I_{s\lambda} & -kw^f u^I_{lc} + u^I_{ll} \\ 0 & \left(kw^f\right)^2 u^I_{cc} - 2kw^f u^I_{cl} + u^I_{ll} \end{bmatrix} > 0.$$

Now, let us focus on the consequences of λ on leisure. We have

$$\begin{aligned} \frac{dl^I}{d\lambda} & = -\left(\frac{ds^I}{d\lambda} + \frac{da^I}{d\lambda}\right) \\ & = -\left(-G^I_{s\lambda}G^I_{aa} + G^I_{s\lambda}G^I_{as}\right) \\ & = kw^f G^I_{s\lambda}\left(\left(kw^f u^I_{cc} - u^I_{cl}\right)\right) \end{aligned}$$

Therefore we have $dl^I/d\lambda \leq 0$ if $u^I_{cl} > 0$.

Similarly, for $j = N$, comparative statics with respect to λ are given by:

$$\frac{ds^N}{d\lambda} = \frac{1}{|G^N|} \begin{bmatrix} -G^N_{s\lambda} & G^N_{sa} \\ 0 & G^N_{aa} \end{bmatrix}$$

and

$$\frac{da^N}{d\lambda} = \frac{1}{|G^N|} \begin{bmatrix} G^N_{ss} & -G^N_{s\lambda} \\ G^N_{as} & 0 \end{bmatrix}.$$

Differentiation with respect to λ gives:

$$\begin{aligned} G^N_{s\lambda} &= -\frac{\frac{\partial p^N(s^N)}{\partial s^N}\phi}{A^2} \left[\left[u(w^f T) - u^N \left(kw^f a^N + z, T - s^N - a^N \right) \right) \right] \left(p^I(s^I) - p^N(s^N) \right) \\ &\quad + \left(r + \phi + p^N(s^N) \right) \left[u^I(kw^f a^I + bw^f H, T - s^I - a^I) - u^N(kw^f a^N + z, T - s^N - a^N) \right]. \end{aligned}$$

Similarly, for $p^I(s^I) \geq p^N(s^N)$, we have $G^N_{s\lambda} < 0$. It implies that $ds^N/d\lambda \leq 0$ (as $G^N_{aa} < 0$) and that

$$\begin{aligned} sign\left(\frac{da^N}{d\lambda}\right) &= -sign\left(G^N_{as}\right) \\ &= sign\left(kw^f u^N_{cl} - u^N_{ll}\right). \end{aligned}$$

Finally, the effect of λ on leisure l^N is determined by:

$$\begin{aligned} \frac{dl^N}{d\lambda} &= -G^N_{s\lambda}\left(G^N_{as} - G^N_{aa}\right) \\ &= kw^f G^N_{s\lambda}\left(\left(kw^f u^N_{cc} - u^N_{cl}\right)\right). \end{aligned}$$

As $G^N_{s\lambda} < 0$, we have

$$sign\left(\frac{dl^N}{d\lambda}\right) = sign\left(u^N_{cl} - kw^f u^N_{cc}\right),$$

which is positive if $u^N_{cl} > 0$. ■